Role of Electrokinetics in the Cleaning-Efficiency of a Dialyzer: Toward an Artificial-Kidney

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Introduction/Motivation

- CKD has no cure, kidney transplants are accompanied by long wait times and the possibility of tissue rejection, while dialysis provides a poor quality of life. [1]
- The goal of this work is to analyze the blood at the **microscopic** level (dialysis level) and then ***downscale*** to a size ideal for potentially designing an artificial kidney. Artificial kidney → better quality of life.
- As the system is downscaled the importance of electrokinetics comes into question in a more detailed manner.

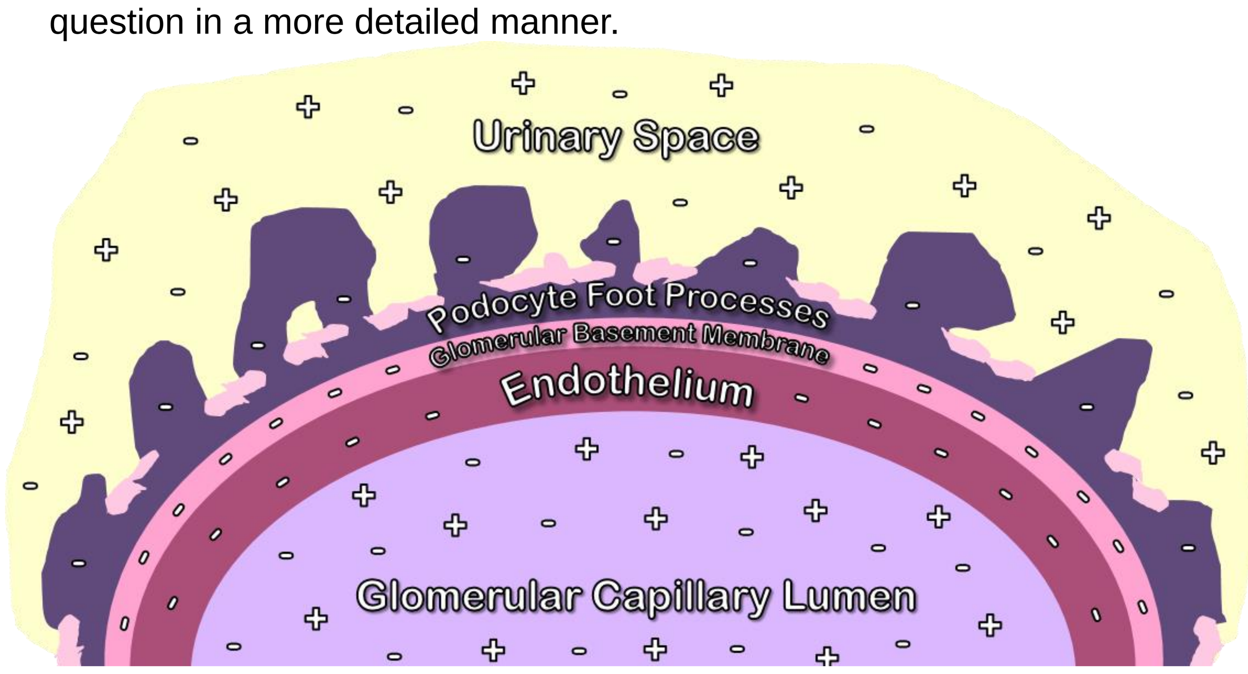

Figure 1: *Layers of Glomerular Capillary* - A glomerular capillary has three main layers of which have various characteristics that determine their filtering specificity due to size **and** charge[2]. Image credit: A. Nastasia Allred

Methods

- A single capillary of cylindrical geometry is used as a potential domain for a "nephron"- the most crucial element of the kidney filtration[2].
- Continuum mechanics approaches are used to analyze the system.

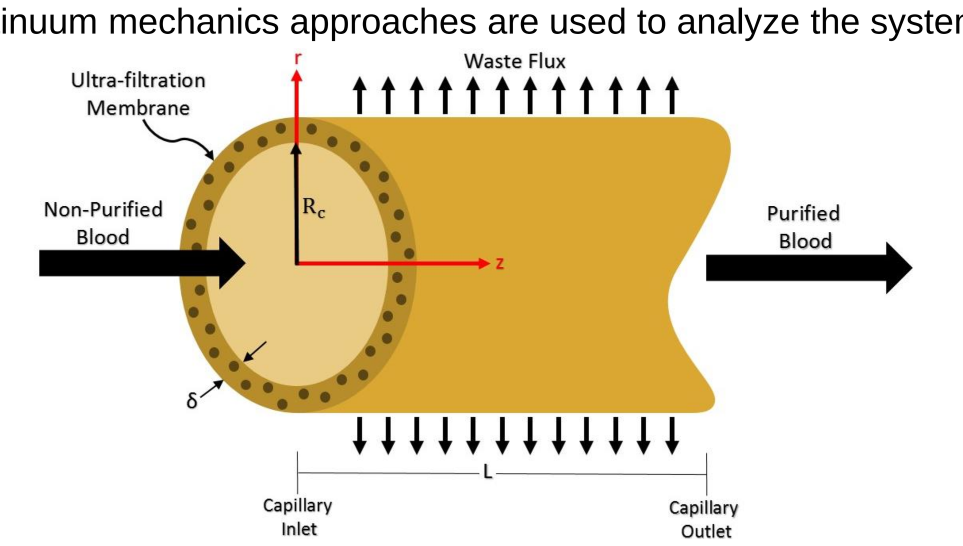

Figure 2: *System Description* - This image shows the system we are describing using our mathematical model and is representative of a glomerular capillary. Image Credit – A. Nastasia Allred.

1. Assumptions

- Newtonian
- Fully developed
- Laminar flow
- Steady State
- Constant $\frac{\partial P}{\partial z}$
- Isothermal
- Incompressible Flow

$$\vec{\nabla}\cdot\vec{v} = 0$$
$$v_r = 0$$
$$v_\theta = 0$$
$$v_z \neq 0$$

Symmetry in θ

$$v_z = v_z(r)$$

2. Model Development

From Navier-Stokes with Electrostatics:

$$\frac{\partial P}{\partial z} = \frac{\mu}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] + \frac{\varepsilon\varepsilon_0 E_z}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right)\right]$$

Poisson-Boltzmann Equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \kappa^2\psi \gg \psi(r) = \frac{\psi_0 I_0(\kappa r)}{I_0(\kappa R_c)}$$

Solution:

$$v_z(r) = \frac{R_c^2}{6\mu}\frac{\partial P}{\partial z}\left(1 - \frac{r^2}{R_c^2}\right) + \frac{\psi_0\varepsilon\varepsilon_0 E_z}{\mu}\left(\frac{I_0(\kappa r)}{I_0(\kappa R_c)} - 1\right)$$

Non-dimensionalized Profile:

$$\hat{v}_z = (1-\rho^2) + \beta\left(\frac{I_0(\hat{\kappa}\rho)}{I_0(\hat{\kappa})} - 1\right)$$

Boundary Conditions

$$\left.\frac{\partial v_z}{\partial r}\right|_{r=0} = 0 \qquad \left.v_z\right|_{r=R_c} = 0$$
$$\left.\frac{\partial \psi}{\partial r}\right|_{r=0} = 0 \qquad \left.\psi\right|_{r=R_c} = \psi_0$$

Dimensionless Numbers

$$\rho = \frac{r}{R_c} \qquad \hat{v}_z = \frac{v_z}{v_z^{max}}$$
$$\beta = \frac{\psi_0\varepsilon\varepsilon_0 E_z}{\mu v_z^{max}}$$
$$\hat{\kappa} = \kappa R_c$$

Methods (Cont.)

3. Solute Portion

$$\frac{\partial C_A}{\partial t} + \vec{\nabla}\cdot\overrightarrow{N_A} = R_A(C_A, T)$$

$$\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right) = (v_z(r) - \mu_A E_z)\frac{\partial C_A}{\partial z}$$

Boundary Conditions

$$\left.-D\frac{\partial C_A}{\partial r}\right|_{r=R_c} = k_g$$
$$\left.\frac{\partial C_A}{\partial r}\right|_{r=0} = 0 \qquad \left.C_A\right|_{z=0} = C_A^o$$

4. Asymptotic Solution

$$\widehat{C_A}(\rho, \hat{z}) = c_0\hat{z} + \Omega(\rho)$$

Global Boundary Condition [3]:

$$\int_0^{R_c}[C_A^0 - C_A(r,z)]\{v_z(r) - \mu_A E_z\}rdr = zk_gR_c$$

$$\int_0^1 -[C_0\hat{z} + \Omega(\rho)]\left\{(1-\rho^2) + \beta\left(\frac{I_0(\hat{\kappa}\rho)}{I_0(\hat{\kappa})} - 1\right) - \gamma\right\}\alpha\rho d\rho = \hat{z}$$

Dimensionless Variables

$$\widehat{C_A} = \frac{C_A - C_A^o}{\frac{k_g R_c}{D}} \qquad \alpha = \frac{v_z^{max}R_c^2}{DL} \qquad \gamma = \frac{\mu_A E_z}{v_z^{max}}$$

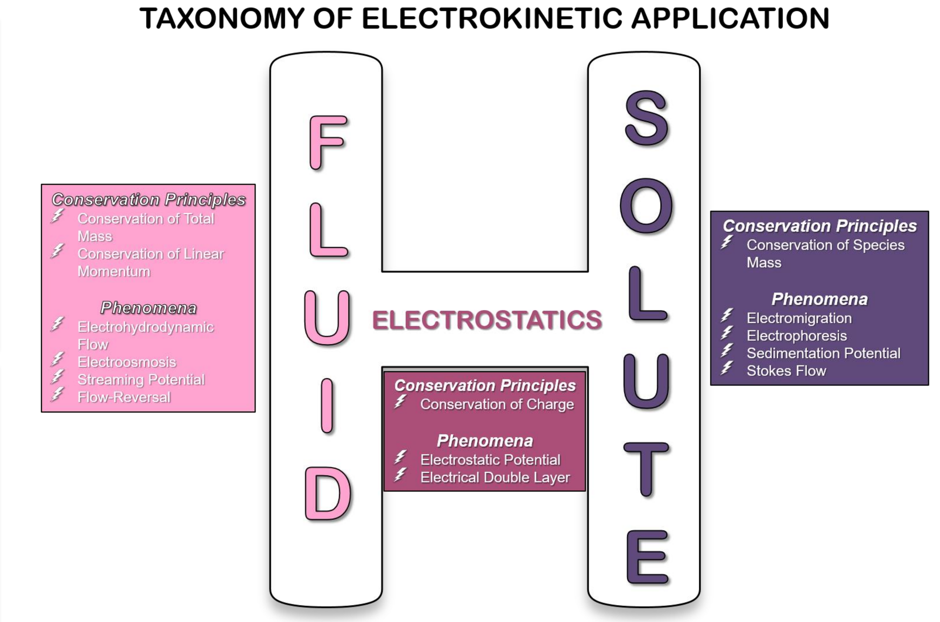

Figure 3: *Taxonomy of Electrokinetic Application* – The "H" is representative of the three main components of Electrokinetic Hydrodynamics. It also shows how the connection of these components results in multiple conservation principles and phenomena[4]. Image credit – A. Nastasia Allred

Results

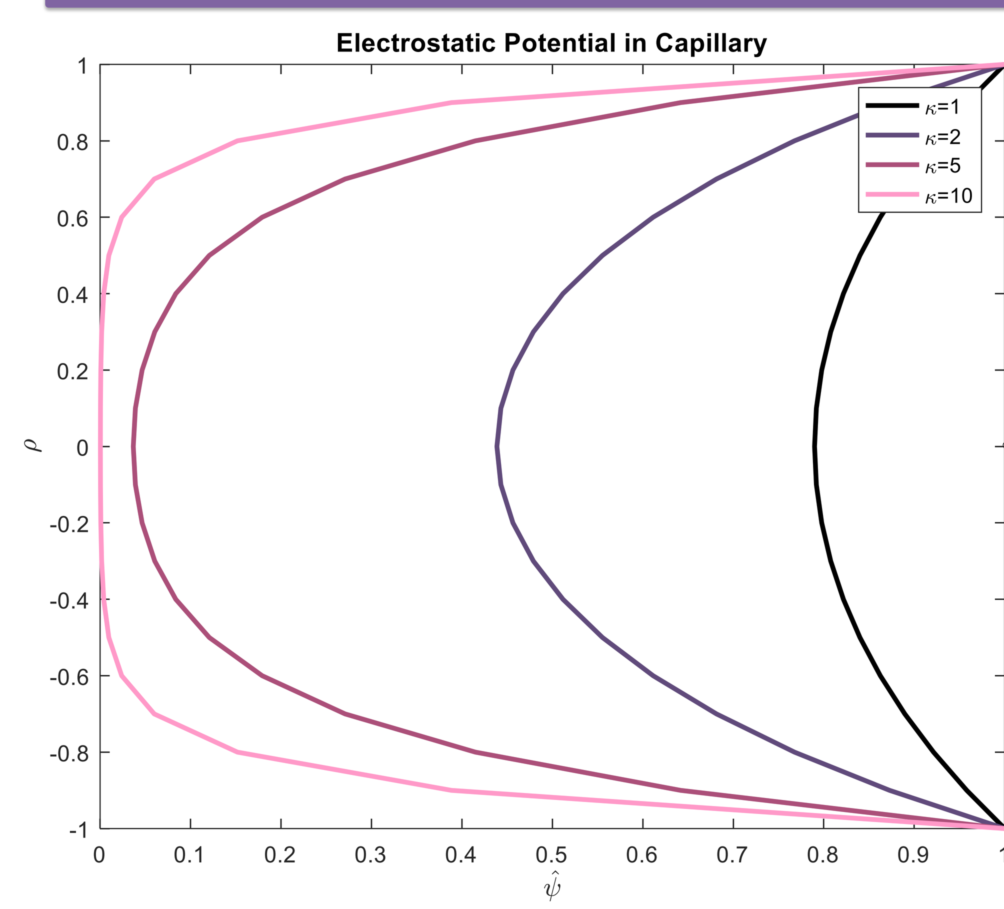

Figure 4: *Electrostatic Potential in Capillary* – This plot shows how the electrostatic potential as a function of non-dimensional radius is affect by changing the kappa parameter which is the inverse Debye length. Image credit – A. Nastasia Allred

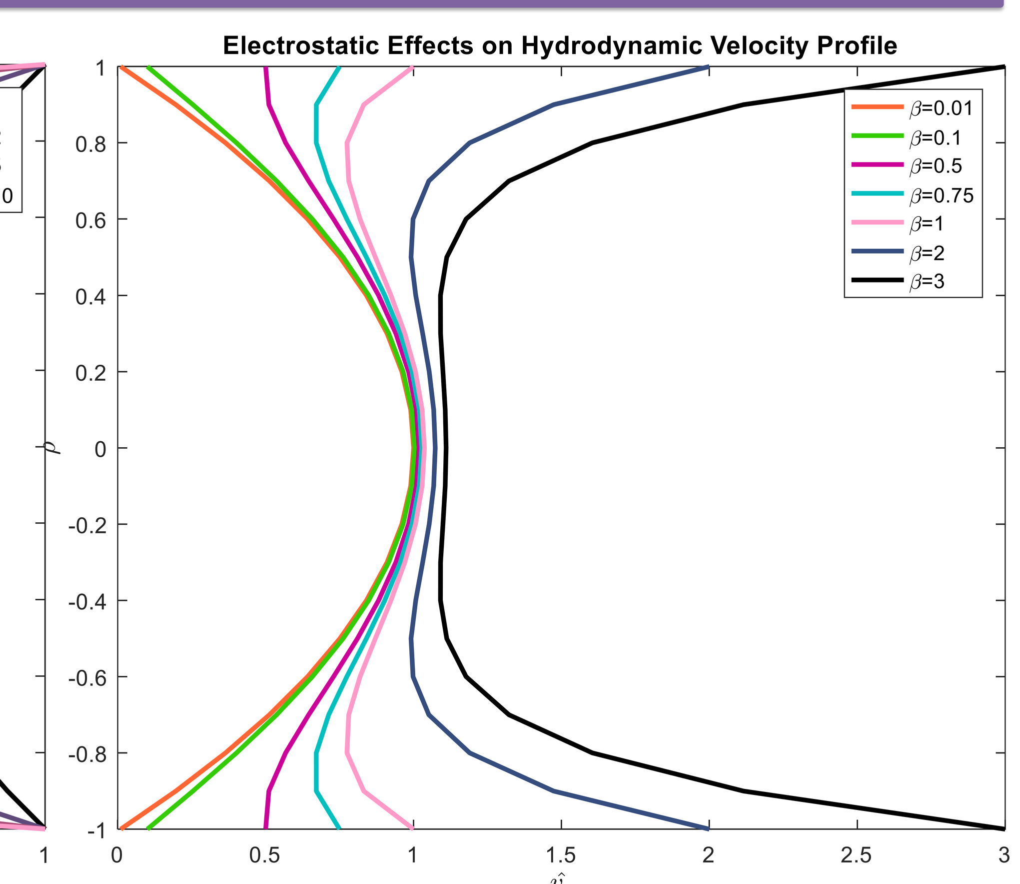

Figure 5: *Electrostatic Effects on Hydrodynamic Velocity Profile* – This image shows how the hydrodynamic velocity profile within the capillary is affected by electrostatic forces. The beta parameter is indicative of the ratio of electrostatic forces to hydrodynamic forces. Image credit – A. Nastasia Allred

Discussion

Beta

$$= \frac{\psi_0\varepsilon\varepsilon_0 E_z}{\mu v_z^{max}}$$

$$= \frac{\text{electrostatic}}{\text{hydrodynamic}}$$

Beta can be used as a scaling/tuning parameter in order to control the separation efficiency of the capillary.

$\beta > 1$	$\beta < 1$	$\beta = 1$
The electrostatic forces are dominating and flow reversal is the result.	The hydrodynamic forces are dominating and the net flow is in the direction of the pressure gradient.	The electrostatic and hydrodynamic forces are equal. We have a balance of flow reversal toward the walls and forward flow in the center.

Conclusions/Future Work

- Currently, progress has been made toward developing a model to depict the electrostatic effects on filtration in the glomerular capillary.
- The electroosmosis profile has been developed and the electrostatic effects have been illustrated. These parameters can be used to increase separation efficiency of the capillary.
- The next step of this research is to develop an asymptotic solution for the concentration profile. The model has been set up, but due to complex functions (such as Bessel functions), computational software including Matlab and Maple will be incorporated.

Global BC
- Use Maple to solve Global BC for integration constants C_0, C_1, C_2
- Put constants into asymptotic solution

Plotting
- Plots will then be developed using Maple and/or Matlab software
- Parameters (α and γ) will be varied in order to optimize the filtration in the capillary.

Validation
- Feasibility regions will be determined for this solution.
- Values will be tested for feasibility.

References

[1]"Chronic Kidney Disease." World Kidney Day, 2017. Web. 02 Apr. 2017. <http://www.worldkidneyday.org/faqs/chronic-kidney-disease/>.

[2]McCance, Kathryn L, and Sue E. Huether. *Pathophysiology: The Biologic Basis for Disease in Adults and Children.* St. Louis: Mosby, 1990. Print.

[3]Bird, R. Byron, Warren E. Stewart, and Edwin N. Lightfoot. Transport Phenomena. 2nd ed. New York: John Wiley & Sons Inc, 2002. Print.

[4]Tijaro-Rojas, J. Pascal, M. A. Oyander and P.E. Arce, "The Acquisition and Transfer Of Knowledge Of Electrokinetic-Hydrodynamics (EKHD) Fundamentals: An Introductory Graduate-Level Course," *Journal of European Engineering Education*, Published on line 03, August (2016).

Acknowledgements

I would like to thank Dr. Pedro Arce, Samantha Blanton, and Dr. Robby Sanders for their help and guidance throughout this project.

I would also like to give thanks to the College of Engineering and the Department of Chemical Engineering for funding this research.