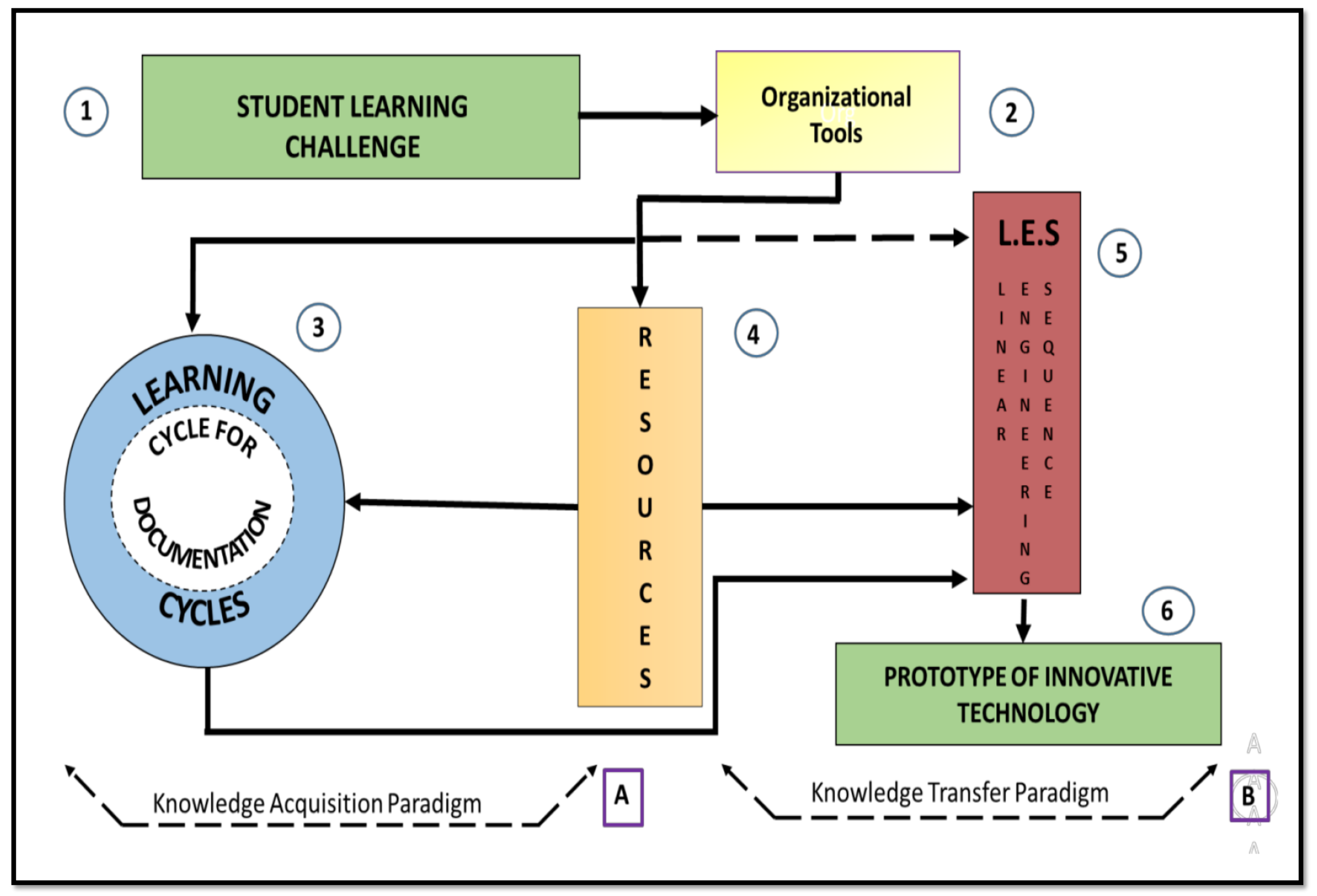
Electrotherapeutic Assisted Wound Healing: Modeling of the Electrostatic Field in a Porous Gel or Healing Media

Phoebe Dawson, Steffano Oyanader, Stephanie Jorgensen, Robby Sanders and Pedro E. Arce
Department of Chemical Engineering; Tennessee Technological University, Cookeville, TN

Motivation and Relevance of Research

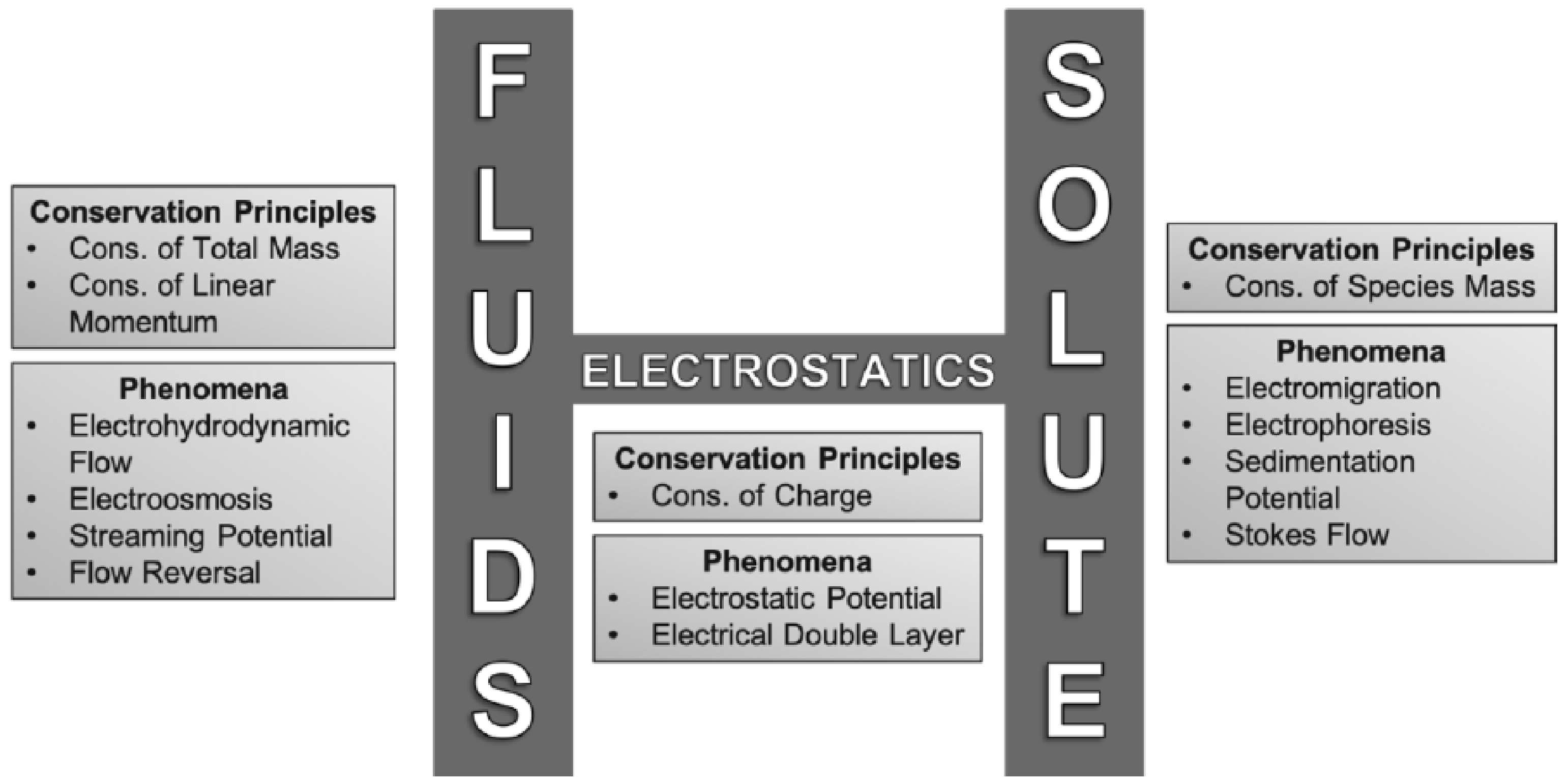
Among the many advances in the biomedical sciences in the last decade, the bio-mathematical foundation to homeostatic wound healing deserves further attention in the scientific community. Recent new contributions (Jorgensen, 2017) have made progress experimentally in understanding transport of biomedicines in hydrogels of potential use as an effective scaffolding material to facilitate wound healing. In addition, work has been done (Oyanader et al, 2020) to increase the understanding of the electro-convective-diffusive transport of biomolecules in wound healing in electrotherapeutic assisted wound healing applications, theoretically. This contribution will focus on the modeling of the electrostatic electrical field effects in the wound microenvironment of the scaffolding material by using idealized pore domains to describe pore morphology. The driving interest of our study is to understand the effects of the electrokinetic forces on the diffusion and migration of thrombin to induce the conversion of fibrinogen to fibrin, as this would be one of the initial steps in the early-phases of the wound healing process. Specifically, the electrostatic Laplace equation, in a pore domain of cylindrical geometry, will be solved via the use of area-averaging methods and its solution will be parametrically illustrated for a set of values of the applied voltages. The role of material, scale, and electro-migration on the transport of bio nutrients and medicines via the use of the molar species continuity equation will be discussed. Future steps in the research project will be highlighted.

Methodology



The methodology for modeling the wound microenvironment shown is based on the Renaissance Foundry (see figure above) [3] and the area averaging approach [5]. A thorough review of each of these works detail how to properly obtain the driving methodology to solve the governing equations brought about from the wound microenvironment. In addition, the Electrokinetic-Hydrodynamics H-model [1] will be used to direct the dynamic model.

Development of Study



FLUIDS

Conservation Principles
- Cons. of Total Mass
- Cons. of Linear Momentum

Phenomena
- Electrohydrodynamic Flow
- Electroosmosis
- Streaming Potential
- Flow Reversal

ELECTROSTATICS

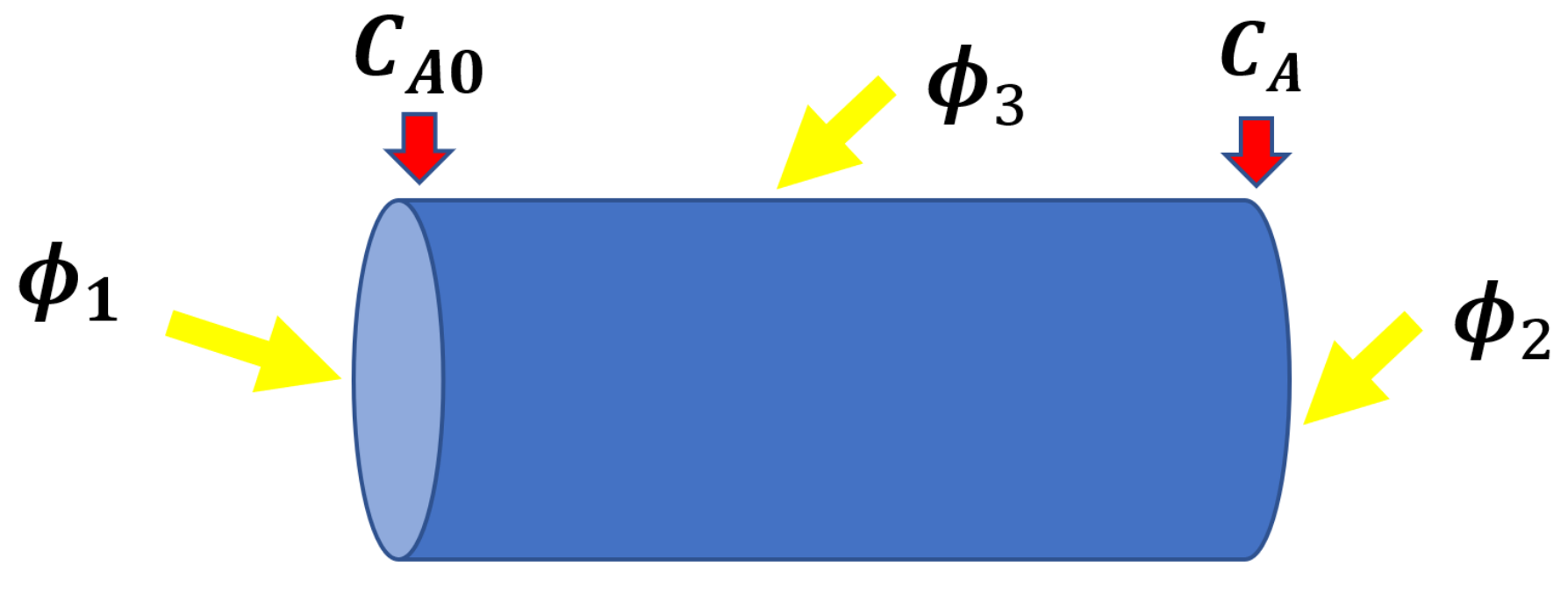
Conservation Principles
- Cons. of Charge

Phenomena
- Electrostatic Potential
- Electrical Double Layer

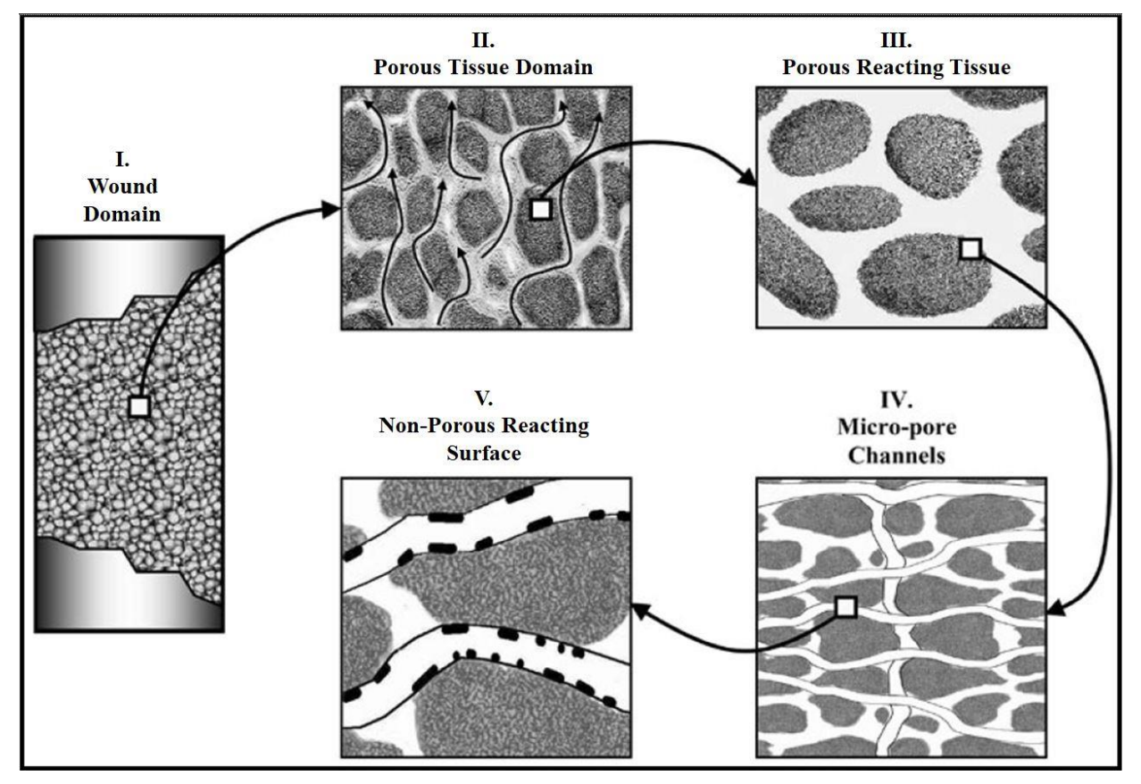
SOLUTE

Conservation Principles
- Cons. of Species Mass

Phenomena
- Electromigration
- Electrophoresis
- Sedimentation Potential
- Stokes Flow



Wound Microenvironment



Conservation of Species Mass

Assumptions	Species Mass Equation
- Constant Density - No Radial Concentration - No Effects of Joule Heating - Electrical Field is applied perpendicularly to the direction of flow	$\frac{\partial C_a}{\partial t} = D\frac{\partial^2 C_A}{\partial z^2} + D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right)\right] + z\mu\frac{\partial}{\partial z}\left[C\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial C_A}{\partial z}\frac{\partial \phi}{\partial z}\right]$ - **C_A**: concentration of species A - **t**: time - **D**: Diffusivity - ϕ: Electroconvection

Boundary Conditions

$$C_A = C_A^0 \; @ \; z = 0 \qquad -D\frac{\partial C_A}{\partial r} = kC_{AS} \; @ \; r = R_0$$

$$\frac{\partial C_A}{\partial z} = 0 \; @ \; z = L \qquad \frac{\partial C_A}{\partial r} = 0 \; @ \; r = 0$$

Conservation of Electrostatic Charge

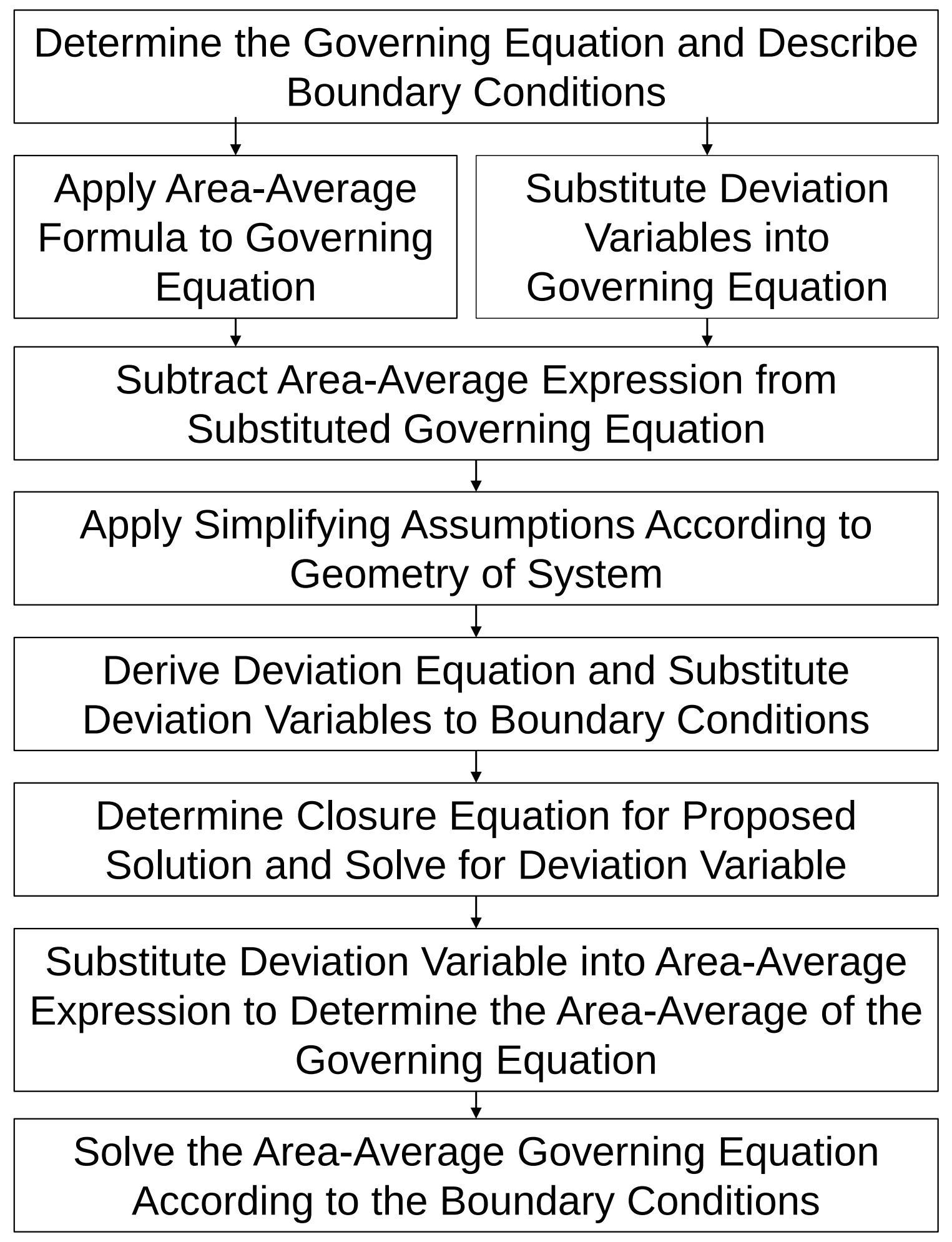
Assumptions	Electrostatic Conservation Equation
- Laplacian Formation - No Radial Effects of the Electrical Field	$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \phi}{\partial r}\right] + \frac{\partial^2 \phi}{\partial z^2} = 0$ - ϕ: Electroconvection

Boundary Conditions

$$\frac{\partial \phi}{\partial r} = 0 \; @ \; r = 0 \qquad \phi(z) = K_3 \; @ \; r = R_0$$

$$\phi(r) = K_1 \; @ \; z = 0 \qquad \phi(r) = K_2 \; @ \; z = L$$

Discussion



Determine the Governing Equation and Describe Boundary Conditions

Apply Area-Average Formula to Governing Equation

Substitute Deviation Variables into Governing Equation

Subtract Area-Average Expression from Substituted Governing Equation

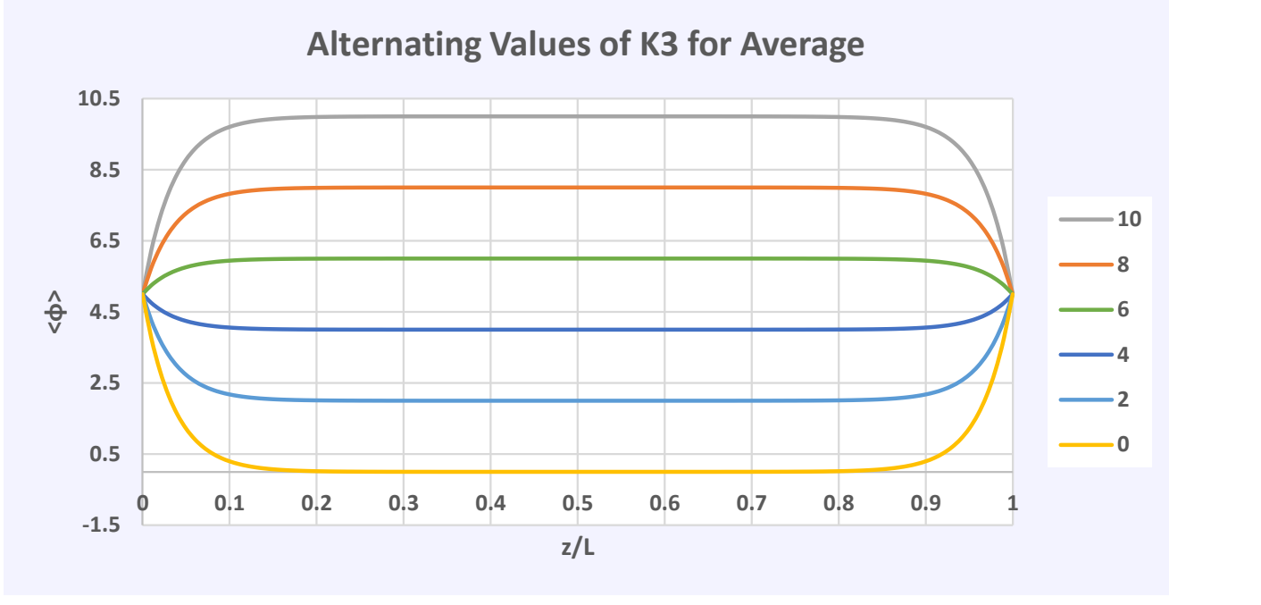
Apply Simplifying Assumptions According to Geometry of System

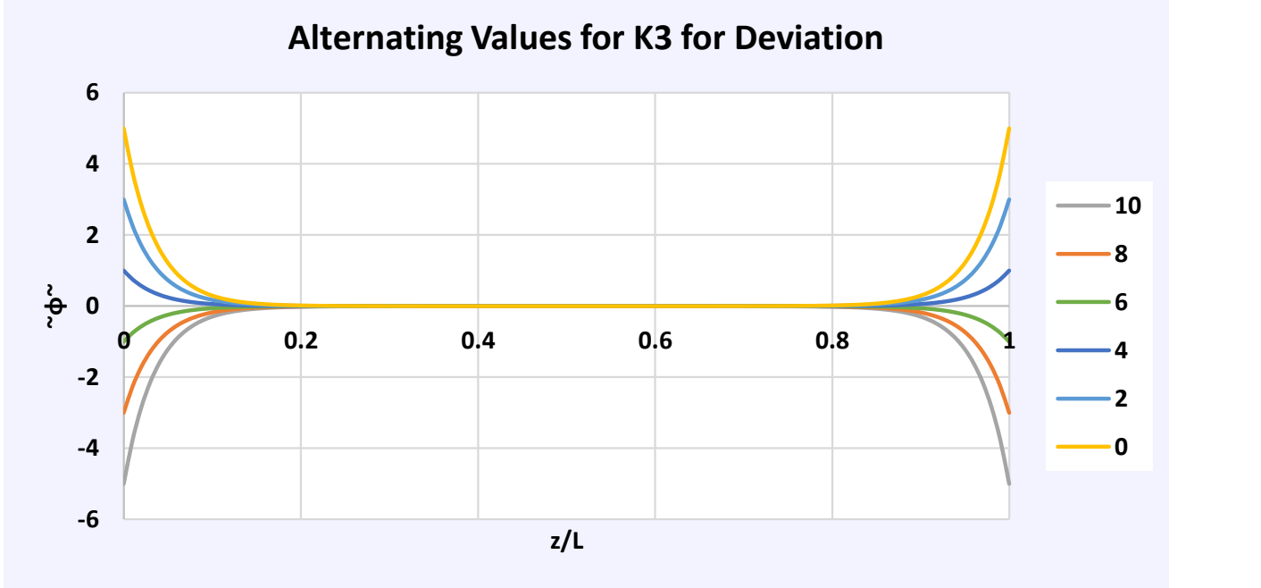
Derive Deviation Equation and Substitute Deviation Variables to Boundary Conditions

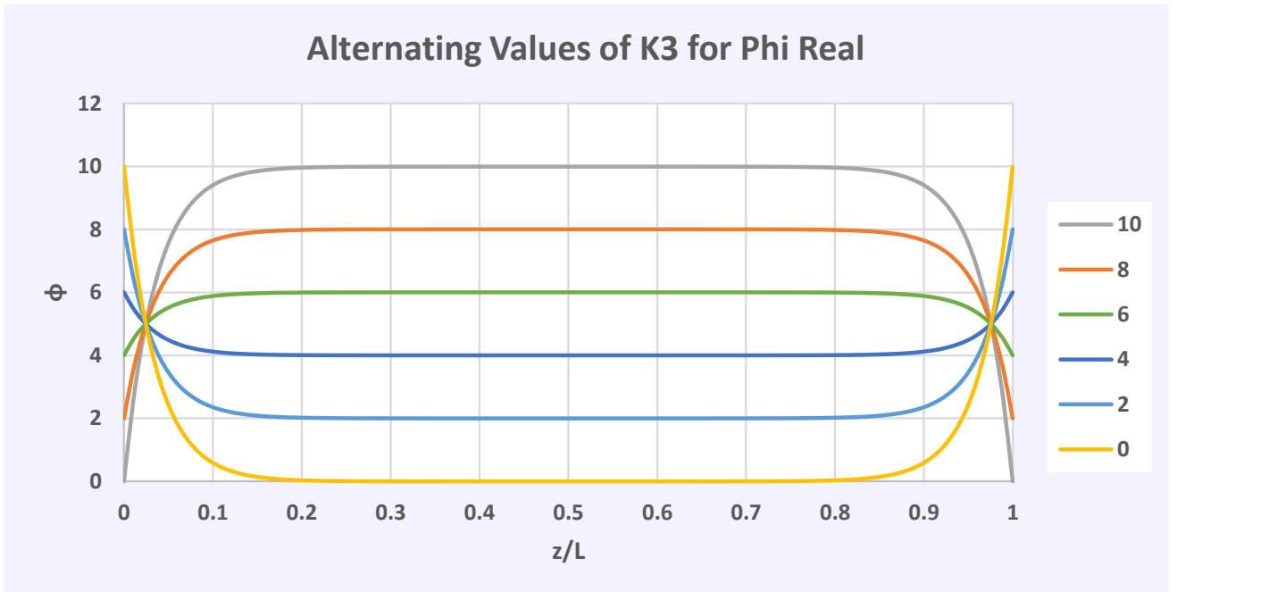
Determine Closure Equation for Proposed Solution and Solve for Deviation Variable

Substitute Deviation Variable into Area-Average Expression to Determine the Area-Average of the Governing Equation

Solve the Area-Average Governing Equation According to the Boundary Conditions







Formal Analytical Solution:

$$\phi = \langle\phi\rangle + \tilde{\phi}$$

$$\tilde{\phi} = K_3 + \frac{2 * K_3}{R_0^2} * (r^2 - R_0^2) - \left[\frac{4}{R_0^2} * \frac{(r^2 - R_0^2)}{2} + 1\right] * \langle\phi\rangle$$

$$\langle\phi\rangle = c_1 \cosh\left(\sqrt{\frac{8}{R_0^2}} * x\right) + c_2 \sinh\left(\sqrt{\frac{8}{R_0^2}} * x\right) + K_3$$

c_1 and c_2 are constants of integration

Conclusions and Future Work

In conclusion, one can visually understand the solutions provided by the algorithmic approach to solving the Laplace Electrostatic Equations. There is a clear outcome shown by changing the electrostatic initial values of the system. Additionally, it can be seen on each of the graphs that between 0.2 and 0.8 non-dimensional length of the system, one can see a pseudo-steady state that will be used to solve the Concentration Profile. The Concentration (C_A) will be studied with the effects of the applied electrical field.

References

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