Adaptive Motion Control of Quadrotors under Parametric Uncertainties with Stability

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Abstract

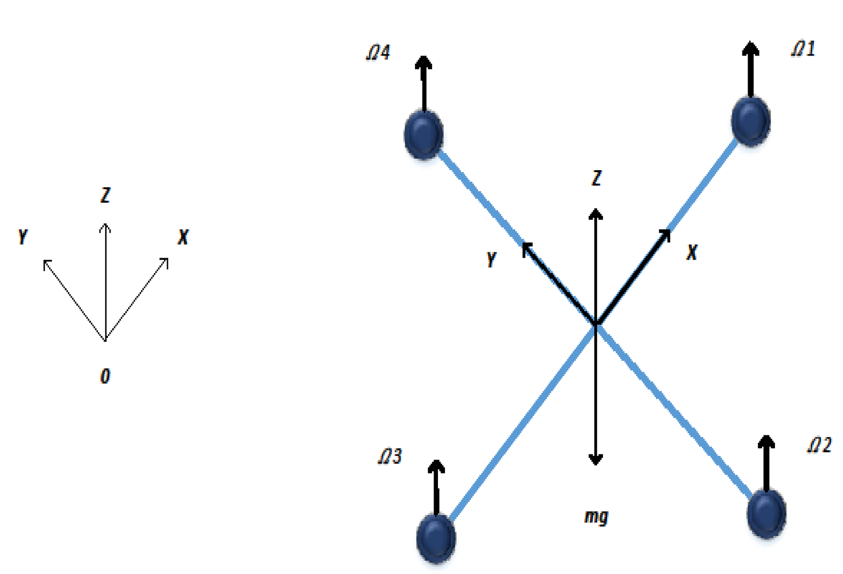
In this poster, an adaptive control strategy is presented for quadrotors under parametric uncertainties. The adaptive control scheme learns the quadrotor's inverse model with a Lyapunov-based adaptation law. For that, a robustness feedback loop is used to stabilize the quadrotor at start-up. Therefore, the controller achieves accurate motion tracking with parametric uncertainties. Unlike many controllers, the proposed adaptive control scheme's stability is guaranteed by Lyapunov direct method. The proposed controller's performance in coping with parameter variations is highlighted in different operating conditions.

Introduction

Unmanned Aerial Vehicles (UAVs) have attracted the attention of the scientific community from diverse disciplines due to their versatile applications. Their popularity has increased exponentially, particularly quadrotors, which is mainly due to their ability to hover and maneuver in tight and dangerous places. Consequently, they have become widely used in many applications such as surveillance, exploration, rescue missions and payload transportation.

Studies have shown that the design of robust controllers for mathematically ill-defined systems that may be subjected to structured and unstructured uncertainties is made possible with computational intelligence tools, such as artificial neural networks and fuzzy logic systems. The approximation capabilities have been the main driving force behind the increasing popularity of such methods as they are theoretically capable of uniformly approximating any continuous real function to any degree of accuracy. This has led to the recent advances in the area of intelligent control. Satisfactory performance is achieved with various neural network models for complex systems control.

The contribution of this research is to achieve motion tracking for quadrotors in the presence of parametric uncertainties. The proposed control scheme makes use of adaptive control theory to cope with parameter variations while a robustness feedback loop copes with the time-varying modeling and disturbance uncertainties. Unlike many controllers, the closed-loop control scheme's stability is guaranteed by Lyapunov direct method. This paper deals with high motion tracking performance under both structured and unstructured uncertainties



Dynamics

$$\ddot{x} = \frac{1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)u_1$$
$$\ddot{y} = \frac{1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)u_1$$
$$\ddot{z} = -g + \frac{1}{m}(\cos\phi\cos\theta)u_1$$
$$\ddot{\phi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} + \frac{J_r}{I_x}\dot{\theta}\Omega_r + \frac{l}{I_x}u_2$$
$$\ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\phi}\dot{\psi} - \frac{J_r}{I_y}\dot{\phi}\Omega_r + \frac{l}{I_y}u_3$$
$$\ddot{\psi} = \frac{I_x - I_y}{I_z}\dot{\theta}\dot{\phi} + \frac{1}{I_z}u_4$$

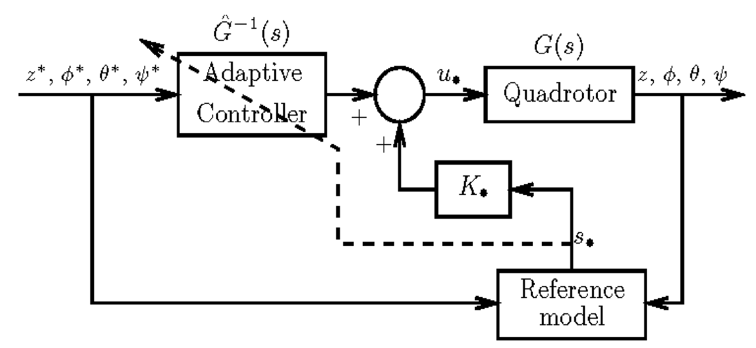
m = mass of the quadrotor
l = length from the rotors to the center of mass
g = gravitational constant
ϕ = roll angle of the quadrotor
θ = pitch angle of the quadrotor
ψ = yaw angle of the quadrotor
u_i = control inputs
J_r = moments of inertia of the propeller blades
Ω_r = angular velocity of the propeller blades
I_x, I_y, I_z = moments of interia of the quadrotor

Where

$$u_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$$
$$u_2 = b(\Omega_2^2 - \Omega_4^2)$$
$$u_3 = b(\Omega_3^2 - \Omega_1^2)$$
$$u_4 = b(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2)$$
$$\Omega_r = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4$$

u_1 provides thrust on the body in the z-axis.
u_2 and u_3 are the roll and pitch inputs.
u_4 is used for the yaw control.
Ω_1, Ω_2, Ω_3, and Ω_4 are speed of the motors

Control Design



Feedforward adaptive controller
A stabilizing feedback
Let $e_z = z - z^*, e_\phi = \phi - \phi^*, e_\theta = \theta - \theta^*, e_\psi = \psi - \psi^*$ denotes the system's tracking errors.
Let's define the following reference model:

$$s_z = \dot{e_z} + \eta_z e_z \quad .....(2a)$$
$$s_\phi = \dot{e_\phi} + \eta_\phi e_\phi \quad .....(2b)$$
$$s_\theta = \dot{e_\theta} + \eta_\theta e_\theta \quad .....(2c)$$
$$s_\psi = \dot{e_\psi} + \eta_\psi e_\psi \quad .....(2d)$$

Where $\eta_\circ$ being a positive constant that defines the desired bandwidth of the closed loop system.
The equation can be formulated as:

$$s_z = \dot{z} - \dot{z}_r \quad .....(3a)$$
$$s_\phi = \dot{\phi} - \dot{\phi}_r \quad ......(3b)$$
$$s_\theta = \dot{\theta} - \dot{\theta}_r \quad ......(3c)$$
$$s_\psi = \dot{\psi} - \dot{\psi}_r \quad ......(3d)$$

Equation 1c, 1d, 1e, and 1f can be rewritten as follows.

$$u_1 = m\frac{\ddot{z}+g}{\cos\phi\cos\theta} \quad ...(4a)$$
$$u_2 = \frac{I_x}{l}\ddot{\phi} - \frac{I_y - I_z}{l}\dot{\theta}\dot{\psi} - \frac{J_r}{l}\dot{\theta}\Omega_r \quad ...(4b)$$
$$u_3 = \frac{I_y}{l}\ddot{\theta} - \frac{I_z - I_x}{l}\dot{\phi}\dot{\psi} + \frac{J_r}{l}\dot{\phi}\Omega_r \quad ...(4c)$$
$$u_4 = I_z\ddot{\psi} - (I_x - I_y)\dot{\theta}\dot{\phi} \quad ...(4d)$$

The desired dynamics can be expressed by the following quadrotor's inverse regression model.

$$m\frac{\ddot{z}_r+g}{\cos\phi\cos\theta} = \Upsilon_z^T P_z \quad ....(5a)$$
$$\frac{I_x}{l}\ddot{\phi}_r - \frac{I_y - I_z}{l}\dot{\theta}\dot{\psi} - \frac{J_r}{l}\dot{\theta}\Omega_r = \Upsilon_\phi^T P_\phi \quad ...(5b)$$
$$\frac{I_y}{l}\ddot{\theta}_r - \frac{I_z - I_x}{l}\dot{\phi}\dot{\psi} + \frac{J_r}{l}\dot{\phi}\Omega_r = \Upsilon_\theta^T P_\theta \quad ....(5c)$$
$$I_z\ddot{\psi}_r - (I_x - I_y)\dot{\theta}\dot{\phi} = \Upsilon_\psi^T P_\psi \quad ...(5d)$$

Equation number 5 can be divided in $\Upsilon_\circ$ which is a vector of known functions (regressor)
$P_\circ$ is a vector of unknown parameters

$$\Upsilon_z = \begin{bmatrix} \frac{\ddot{z}_r}{\cos\phi\cos\theta} \\ \frac{1}{\cos\phi\cos\theta} \end{bmatrix}, \quad P_z = \begin{bmatrix} m \\ mg \end{bmatrix} \quad .... \quad (6a)$$
$$\Upsilon_\phi = \begin{bmatrix} \ddot{\phi}_r \\ -\dot{\theta}\dot{\phi} \\ -\Omega_r\dot{\theta} \end{bmatrix}, \quad P_\phi = \begin{bmatrix} \frac{I_x}{l} \\ \frac{I_y - I_z}{l} \\ \frac{J}{l} \end{bmatrix} \quad ....(6b)$$
$$\Upsilon_\theta = \begin{bmatrix} \ddot{\theta}_r \\ -\dot{\psi}\dot{\phi} \\ -\Omega_r\dot{\phi} \end{bmatrix}, \quad P_\theta = \begin{bmatrix} \frac{I_y}{l} \\ \frac{I_z - I_x}{l} \\ \frac{J}{l} \end{bmatrix} \quad ....(6c)$$
$$\Upsilon_\psi = \begin{bmatrix} \ddot{\psi}_r \\ -\dot{\theta}\dot{\phi} \end{bmatrix}, \quad P_\psi = [\begin{matrix} I_z \\ I_x - I_y \end{matrix}] \quad ....(6d)$$

The control law is defined as,

$$u_1 = \Upsilon_z^T \widehat{P_z} - K_z s_z \quad ....(7a)$$
$$u_2 = \Upsilon_\phi^T \widehat{P_\phi} - K_\phi s_\phi \quad ....(7b)$$
$$u_3 = \Upsilon_\theta^T \widehat{P_\theta} - K_\theta s_\theta \quad ....(7c)$$
$$u_4 = \Upsilon_\psi^T \widehat{P_\psi} - K_\psi s_\psi \quad .....(7d)$$

Where, $K_\circ$ is the gain of a proportional derivative compensator.
$\widehat{P}_\circ$ is the estimate vector.

Adaption Law

Consider a nonlinear system in the form (2.1)-(2.2) with the control law (3.7). The closed loop system's stability is guaranteed with the following adaption law:

$$\dot{\widehat{P_z}} = -\Gamma_z \Upsilon_z s_z$$
$$\dot{\widehat{P_\phi}} = -\Gamma_\phi \Upsilon_\phi s_\phi$$
$$\dot{\widehat{P_\theta}} = -\Gamma_\theta \Upsilon_\theta s_\theta$$
$$\dot{\widehat{P_\psi}} = -\Gamma_\psi \Upsilon_\psi s_\psi$$

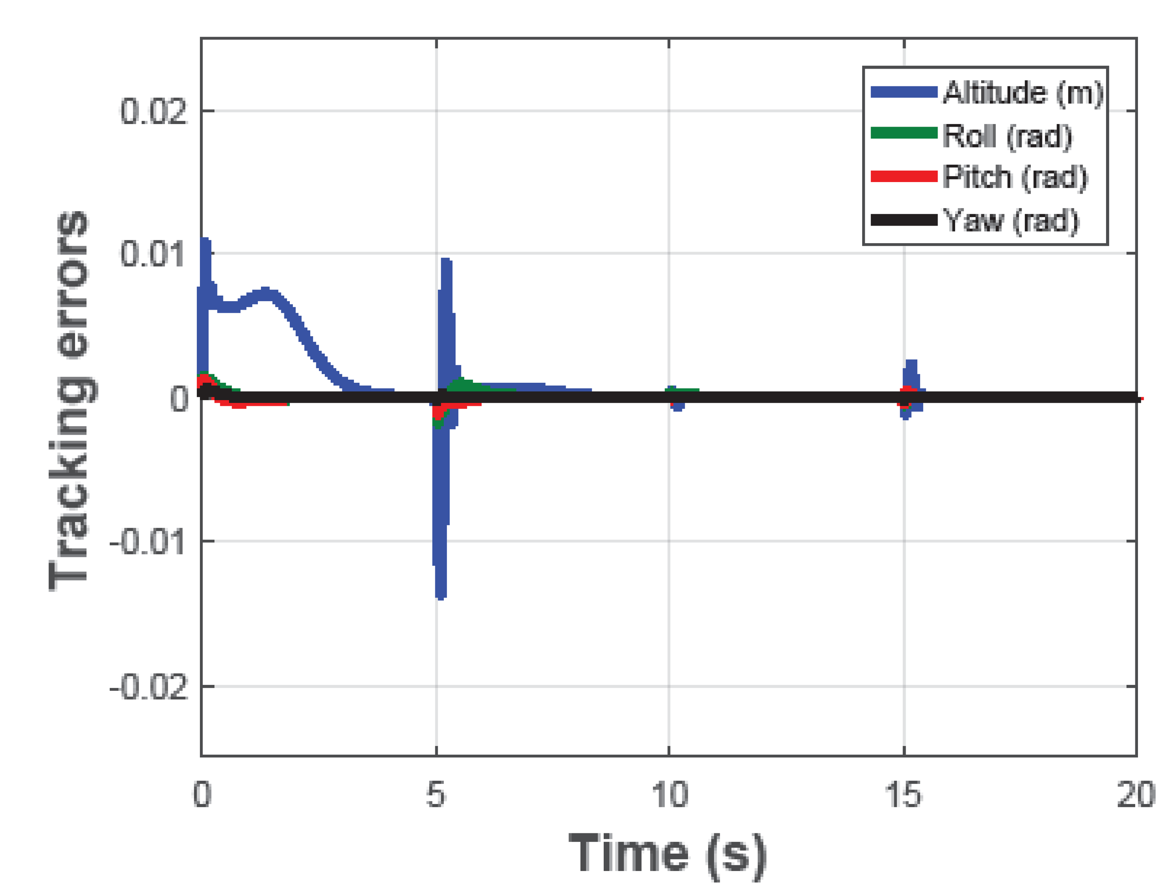
Where $\Gamma = [\gamma_1, \gamma_2, \dots \dots . \gamma_m]^T$ and γ_i is a positive constant gain, $i = 1, \dots \dots m$, with m being the dimension of γ_o or P_o.

Results and Discussion

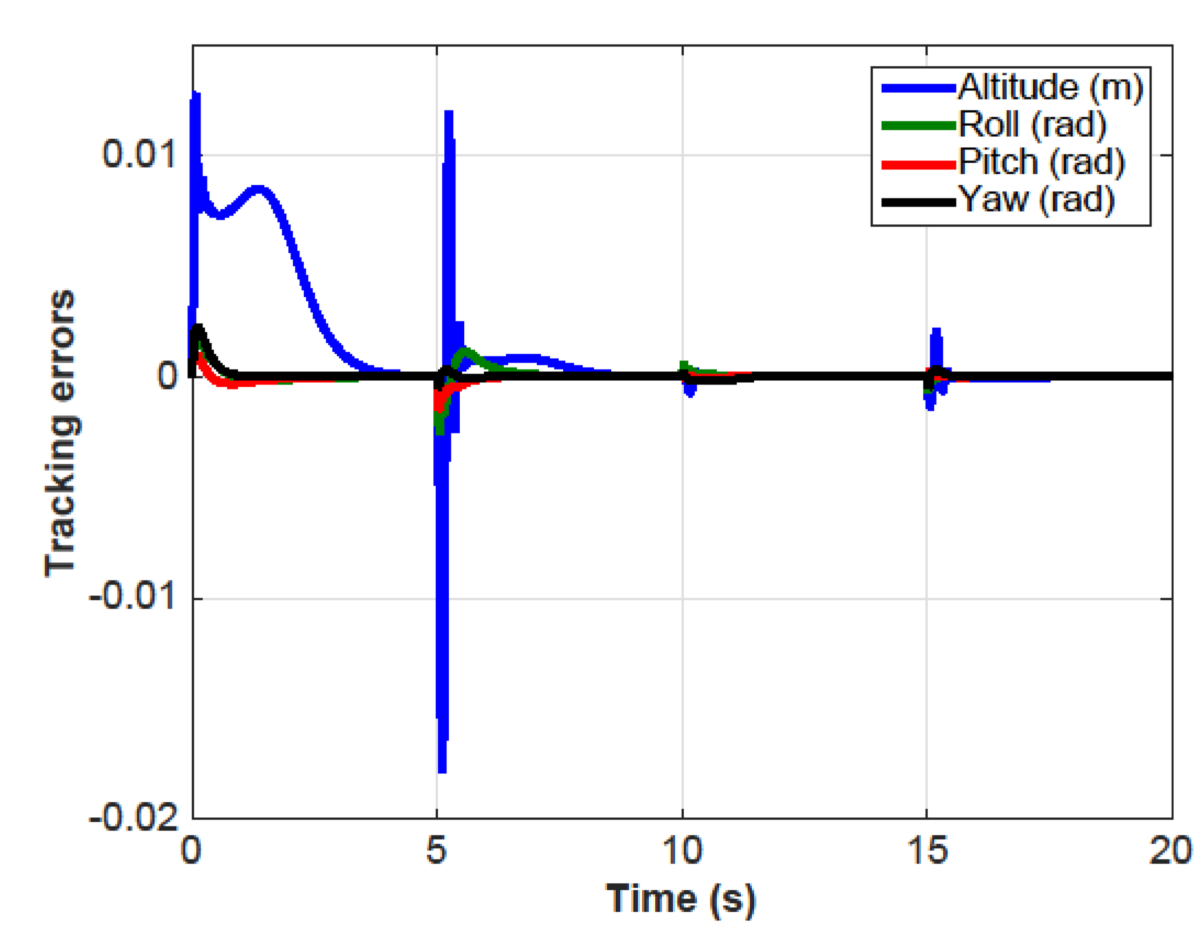
Table 1. Physical parameters of several quadrotors

Parameter	Quadrotor 1	Quadrotor 2	Quadrotor 3	Unit
Mass	0.68	0.80	1.50	(kg)
Arm length	0.18	0.60	1	(m)
Thrust coefficient	$6.9 \cdot 10^{-6}$	$192 \cdot 10^{-7}$	$1.48 \cdot 10^{-7}$	(N·s²)
Drag coefficient	$9.3 \cdot 10^{-8}$	$4 \cdot 10^{-8}$	$2.9 \cdot 10^{-9}$	(N·m·s²)
Rotor inertia	$1.4 \cdot 10^{-5}$	$6 \cdot 10^{-5}$	$2 \cdot 10^{-4}$	(kg·m²)
Inertia on x axis	$3.5 \cdot 10^{-3}$	$15.67 \cdot 10^{-3}$	$8.5 \cdot 10^{-3}$	(kg·m²)
Inertia on y axis	$4.2 \cdot 10^{-3}$	$15.67 \cdot 10^{-3}$	$8.5 \cdot 10^{-3}$	(kg·m²)
Inertia on z axis	$7.5 \cdot 10^{-3}$	$28.34 \cdot 10^{-3}$	$16.33 \cdot 10^{-3}$	(kg·m²)

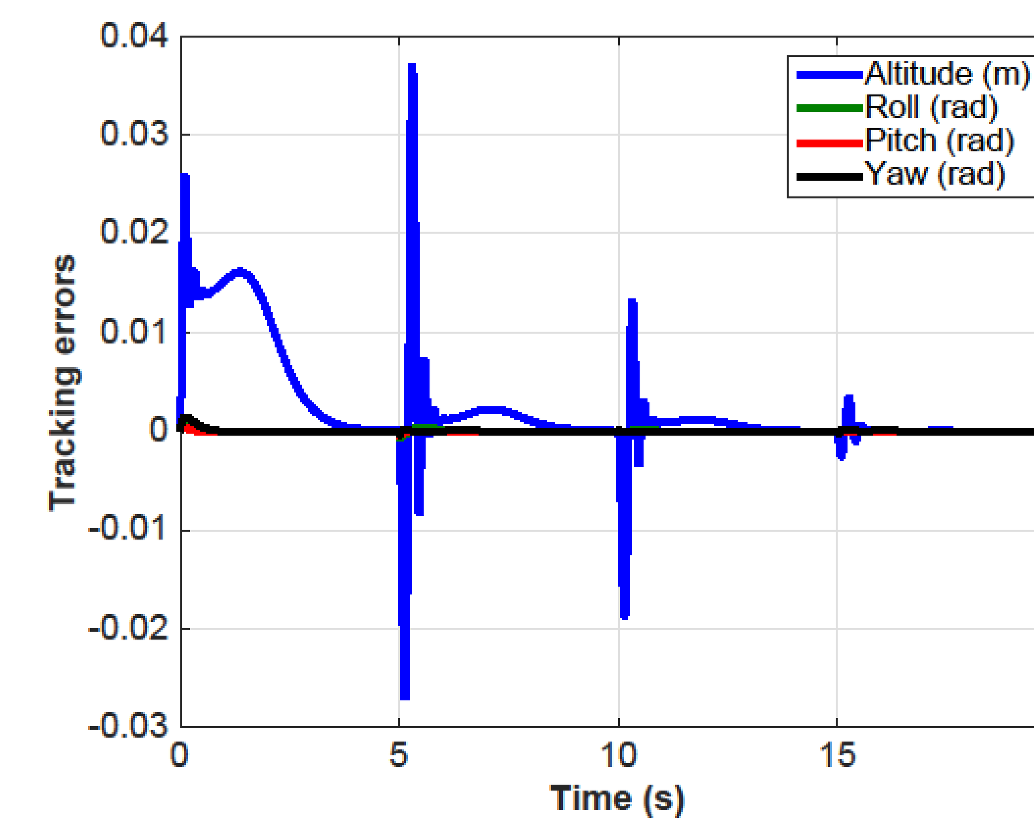
Nominal Case Quadrotor 1



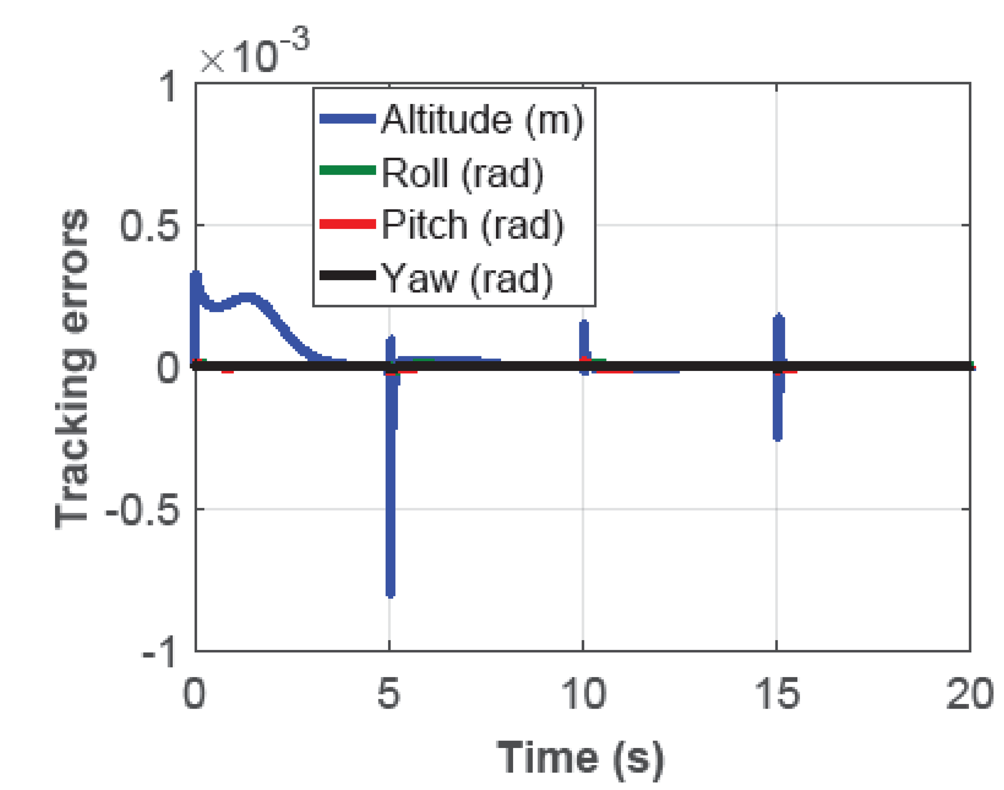
Nominal Case Quadrotor 2



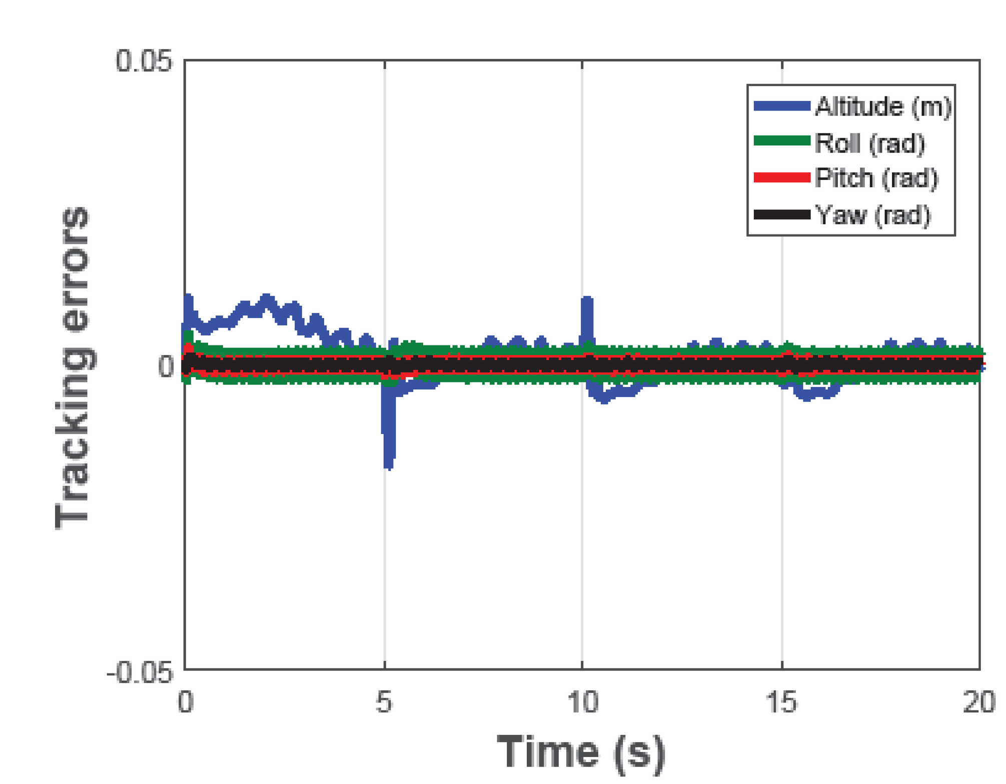
Nominal Case Quadrotor 3



Control under disturbances



Control under noise



A Lyapunov-based adaptive controller is proposed for high-performance control of quadrotors. Results shows that the proposed adaptive controller is more efficient in dealing with parametric uncertainties

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