Investigation of Coinciding Orthogonal Two-Dimensional Structure-Borne Traveling Waves

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Introduction

What are traveling waves?

When a continuous structure is excited at some frequency the structure will deform into a characteristic shape. When a structure is excited at one of its natural frequencies this deflection shape is referred to as a mode shape. These mode shapes are a standing wave in a structure. Standing waves will oscillate in and out-of-plane, but they will not propagate. Traveling waves are similar to standing waves, except that they propagate along the medium they are active in. This concept is easier to visualize in an image, so consider Figure 1.

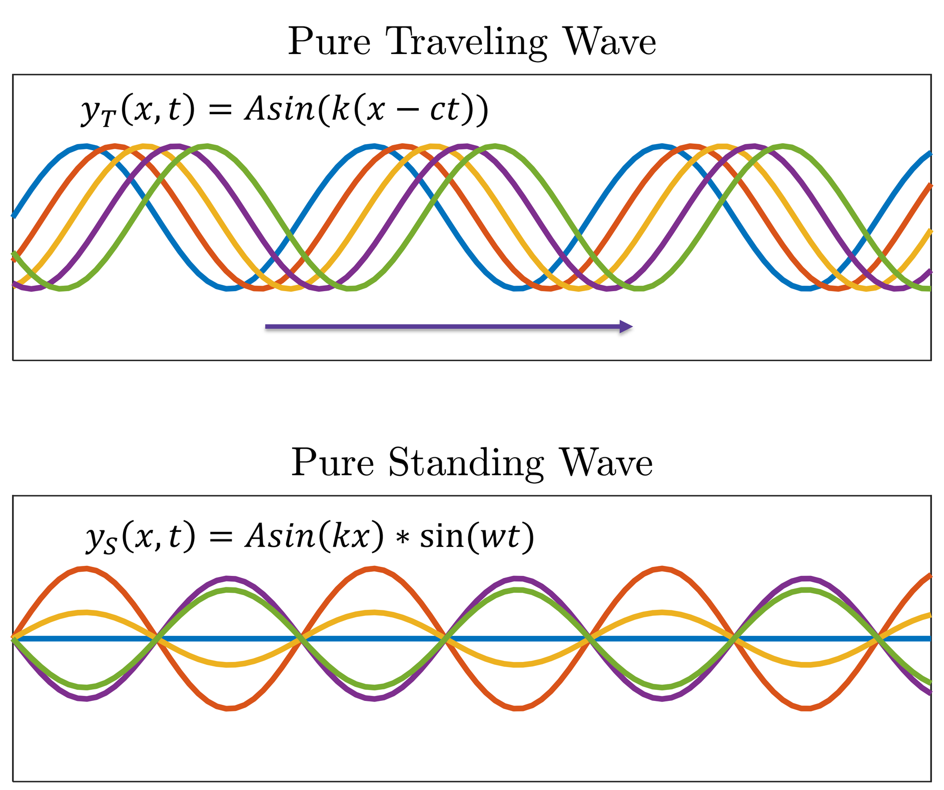

Figure 1: Example of standing and traveling waves. Ordered: blue, red, yellow, purple, green.

Traveling waves are present in nature and can be seen in the swimming behavior of some fish and rays [1]. The propagation of the traveling wave in the wings of a string ray can be seen in Figure 2. An example of standing waves being active in a plate can be seen if Figure 3. The plate has two piezo actuators (the orange colored wafers) that excite the plate at certain frequencies, creating different 2D standing wave patterns in the salt that is setting on the surface of the plate.

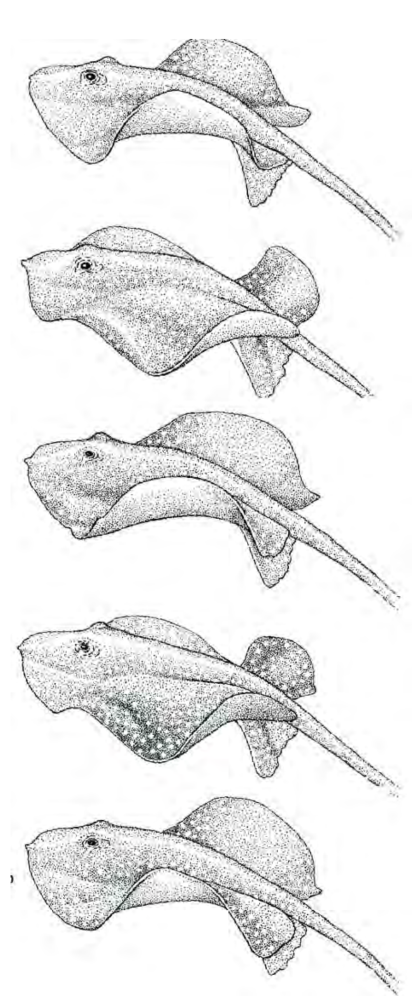

Figure 2: Illustration of a sting ray swimming (Blevins [8]).

Figure 3: Mode shapes shown in lines of salt on a real plate. (a) 253.03 Hz, (b) 315.39 Hz (c) 619.45 Hz

What can we do with traveling waves?

Traveling waves have been used to drive propulsion in fluids [2,3], drag reduction [4], solid-state motion [5], and particle motion applications [6,7]. In the case of particle motion, 2D orthogonal traveling waves are of interest since they have potential as a method of controlled particle motion in 2D on the surface of the plate. We want to investigate the superposition of coinciding orthogonal traveling waves to see if the separate 2D waves combine into waves that can be tailored to propagate in any prescribed direction or if there will only be destructive interference between the two traveling waves.

Methods

How do we generate traveling waves?

Traveling waves can be generated using several different methods. For the purposes of this investigation, we use two-mode excitation. Two-mode excitation uses the modal properties of continuous structures to generate steady-state structure-borne traveling waves (SBTWs). This is done by exciting the structure with two different actuators operating at the same frequency but with a phase difference between them [9].

To generate orthogonal traveling waves using this method, we used two separate pairs of actuators. One pair will excite SBTWs in the crosswise direction (y-axis) and the other pair excites in the lengthwise (x-axis) direction, as shown in Figure 4.

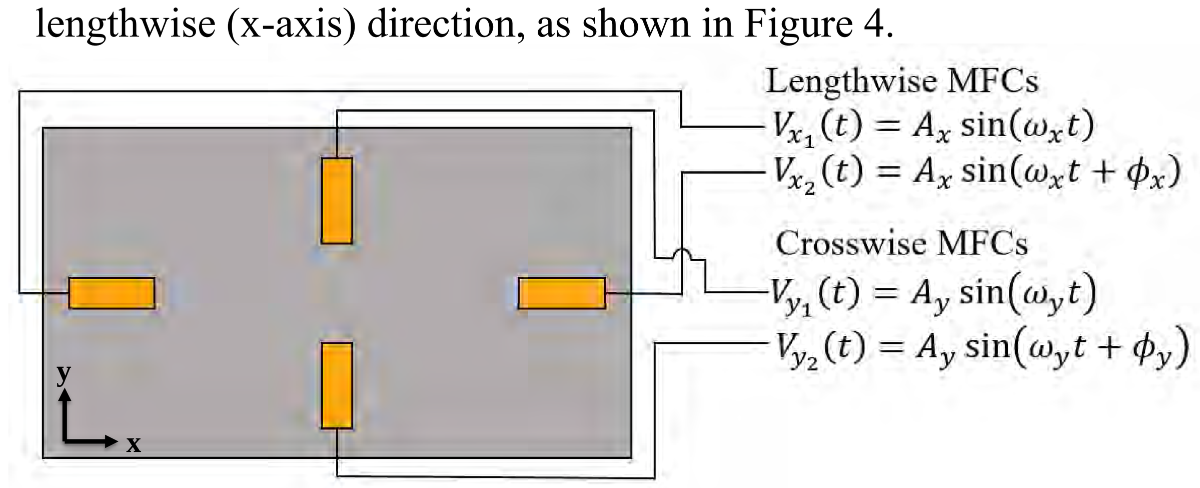

Figure 4: Illustration of the plate with four piezoelectric actuators.

The plate is given dimensions 279.4×584.2×0.7937 mm (11×23 ×1/32 in) and it is modeled as 6061 aluminum with boundary conditions that are assumed to be perfectly fixed on all four sides. The piezoelectric wafers are assumed to be perfectly bonded to the surface of the plate and are modeled as monolithic piezoelectric actuators operating in the 3-1 mode [10]. The Finite element model discretizes the plate into a 19×35 quadratic finite element mesh (seen in Figure 5) using first order shear deformation theory to ensure that it captures the in-plane rotations that are critical to accurately model the high frequency behavior of thin plates.

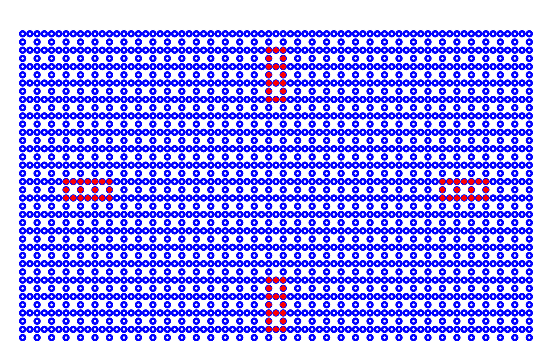

Figure 5: Finite element model of the plate.

After we have excited SBTWs in the plate, we need to address the quality of these traveling waves. For particle motion applications, assessing the quality of the SBTWs is important because the particles will not move in the direction of the SBTW if it is not active where the particles are.

For this investigation, we want to look at two orthogonal SBTWs that are high quality when they are excited separately, and then we will examine how the combination of the two will affect the quality of the superimposed SBTWs. The quality of the SBTWs is evaluated using complex orthogonal decomposition (COD). COD can be used to decompose a complex displacement matrix into a complex correlation matrix [$\boldsymbol{R}$], whose eigenvectors are the complex modes and eigenvalues are the root mean square amplitudes of the complex modes [11]. The complex correlation matrix derived to be

$$\boldsymbol{R} = \frac{1}{N}\boldsymbol{Z}\overline{\boldsymbol{Z}}^T$$

where $\boldsymbol{Z}$ is the matrix of complex motion in the plate where $z_j = [z_j(t_1), \ldots, z_j(t_N)]^T$. Using these vectors, z_j, we can populate the $M \times N$ complex displacement matrix $\boldsymbol{Z} = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_M]^T$. Let the first eigenvector of $\boldsymbol{R}$ be $\boldsymbol{V}_1$, then the traveling index is defined:

$$T_i = \frac{1}{cond([real(\boldsymbol{V}_1), imag(\boldsymbol{V}_1)])}$$

A traveling index of $T_i \approx 0$ corresponds to a pure standing wave and $T_i \approx 1$ corresponds to a pure traveling wave [10,11].

Results

Investigation into Coinciding Orthogonal SBTWs

First, we solved the Finite Element model's equation of motion to find its frequency response function (FRF) of the plate from 0-1000 Hz for both the x-axis and y-axis MFCs.

Note, that not all of the peaks between the x and y axis FRF plots overlap. This is because the different actuator locations are able to activate some natural frequencies that others cannot.

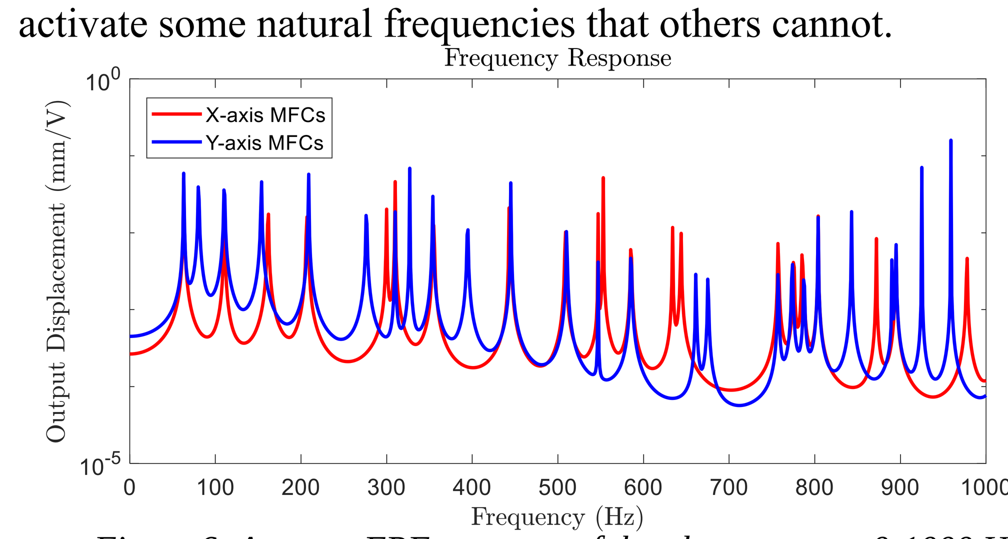

Figure 6: Average FRF response of the plate across a 0-1000 Hz range.

Each of the peaks in the FRF correspond to a 2D mode shape that can be excited in the plate. The first twenty-four mode shapes for this plate can be seen in Figure 8. Blue corresponds to the negative deflection and red to the positive deflection. With green being the median value. For all modes except 14 and 15 green represents nodal lines where there is zero deflection.

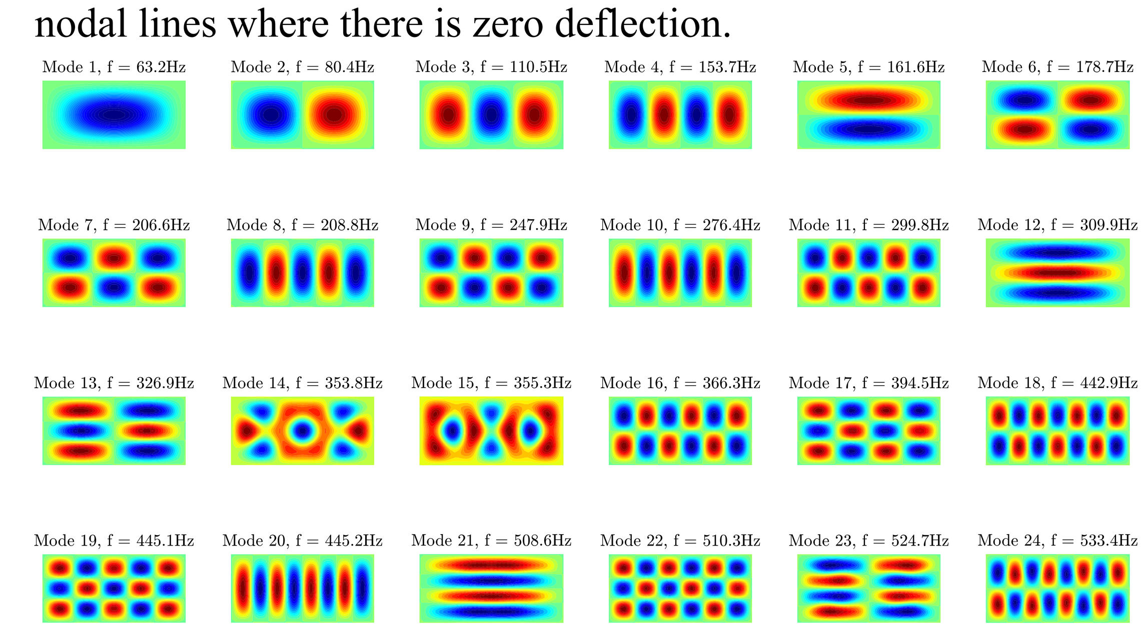

Figure 8: First 24 mode shapes of the plate.

SBTWs generated using two-mode excitation have displacement fields that are primarily made up of a combination of the two adjacent mode shapes that occur at the natural frequencies above and below the excitation frequency. Which mode shapes combine depends on the frequency used and the position of the actuators. The frequencies selected for this investigation are listed in Table 1.

Table 1: Actuation configurations excited in the plate.

SBTW Excitation Cases				
	Freq. (Hz)	Phase (deg)	T_i	Modes
X-axis	470	110	0.94	20 + 22
Y-axis	221	67	0.96	8 +12

Because of the actuator locations, there are specific modes that cannot be excited in the plate with the chosen set of actuators. For example, the x-axis actuators are unable to excite mode 21. This allows for modes 20 and 22 to be combined to form the x-propagating SBTW. This is also the case with the y- propagating SBTW, since the actuators cannot excite modes 9 or 10, allowing the combination of modes 8 and 11. To superimpose the SBTWs, we follow the methods of previous work that has confirmed the superposition of voltage excitation to the piezo actuators will generate SBTWs with superimposed displacement fields of the plate from the individual frequencies [12].

Discussion

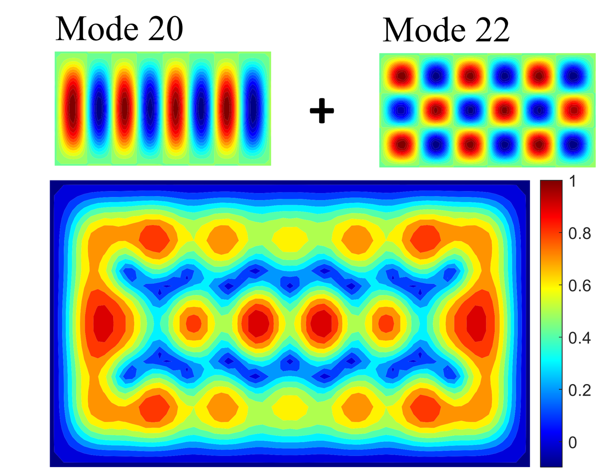

Figure 8: Contributing mode shapes and the normalized RMS velocity of the plate for the x-axis configuration.

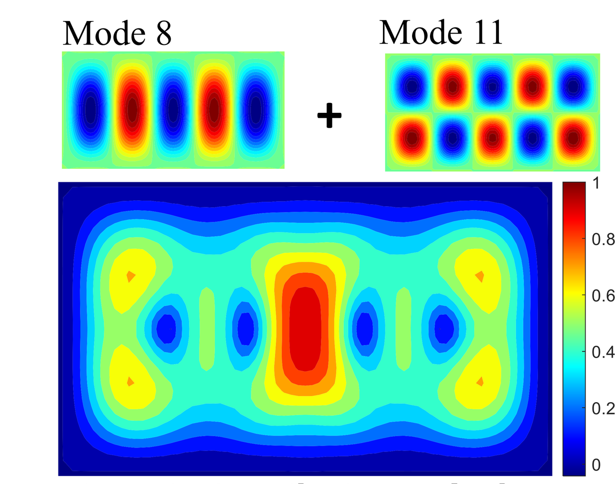

Figure 9: Contributing mode shapes and the normalized RMS velocity of the plate for the y-axis configuration.

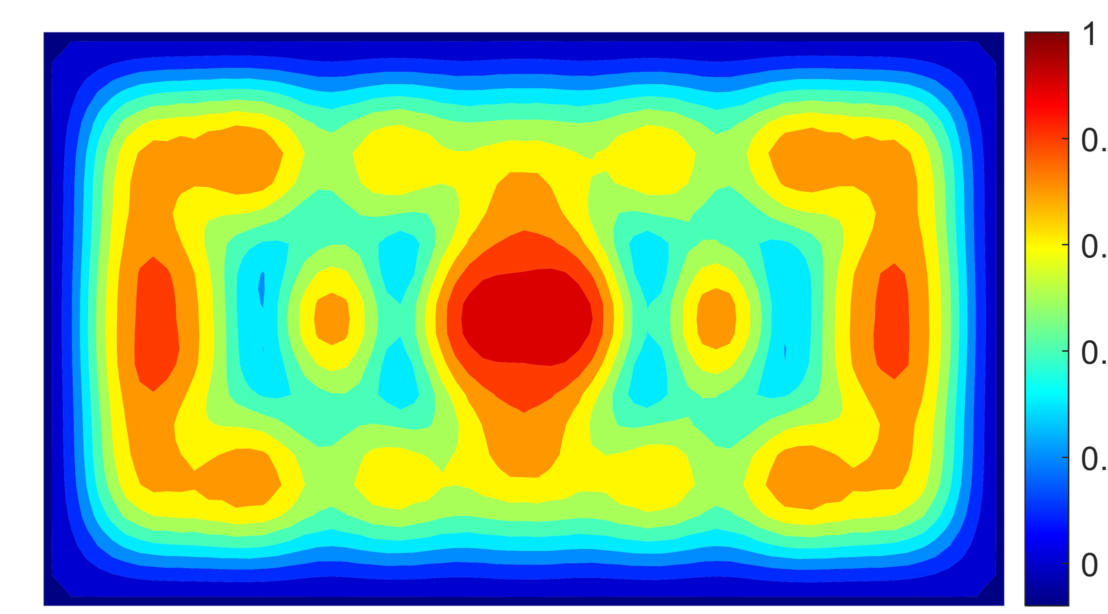

Figure 10: Normalized RMS velocity of the plate for the superimposed signal. $T_i \approx 0.96$.

These frequencies were selected for this investigation because the mode shapes contributing to each orthogonal SBTW are different and far apart, so that the interference between the resulting SBTWs will be minimized. Figures 8 and 9 show the RMS velocity across the entire plate with the individual sets of actuators excited at the frequencies prescribed in Table 1. Figure 10 shows the RMS plot of the superimposed excitations. From the RMS plot alone, it is not clear which direction propagation the SBTWs are traveling in. Applying COD to the combined case we get $T_i \approx 0.96$. This confirms that these signals can combine constructively to produced an SBTW that does still propagate across the plate. A MATLAB code was written to animate the propagation of the SBTW, but observing the motion it still was not clear if the SBTW was propagating primarily in a diagonal direction. Therefore, we can conclude that the SBTW is propagating but not how much it is propagating in a given direction.

Conclusion

A Finite Element model of a 2D plate was developed with two pairs of piezoelectric actuators bonded to its surface. This plate was excited to generate 2D SBTWs using two-mode excitation. This investigation has shown that orthogonal SBTWs can coincide and be tailored to generate SBTWs that maintain their high quality traveling indices. There are still questions to ask about the 2D SBTWs beyond COD. COD guarantees us that the superimposed SBTWs will propagate, but COD is not sensitive to the direction of propagation. Future work will need to scrutinize the quality of the SBTWs traveling in a given direction on the plate. Application of such a quality metric will be the next step in tailoring orthogonal SBTWs to be superimposed with the intention of generating a high quality SBTW in any direction across the surface of the plate.

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