Electrotherapeutic Assisted Wound Healing: Comparison of the Electrostatic Potential in Porous Gel or Healing Media in Cartesian and Cylindrical Geometries

Student Researchers: Phoebe Dawson and Steffano Oyanader

Faculty Mentors: Dr. Pedro E. Arce and Dr. J. Robby Sanders

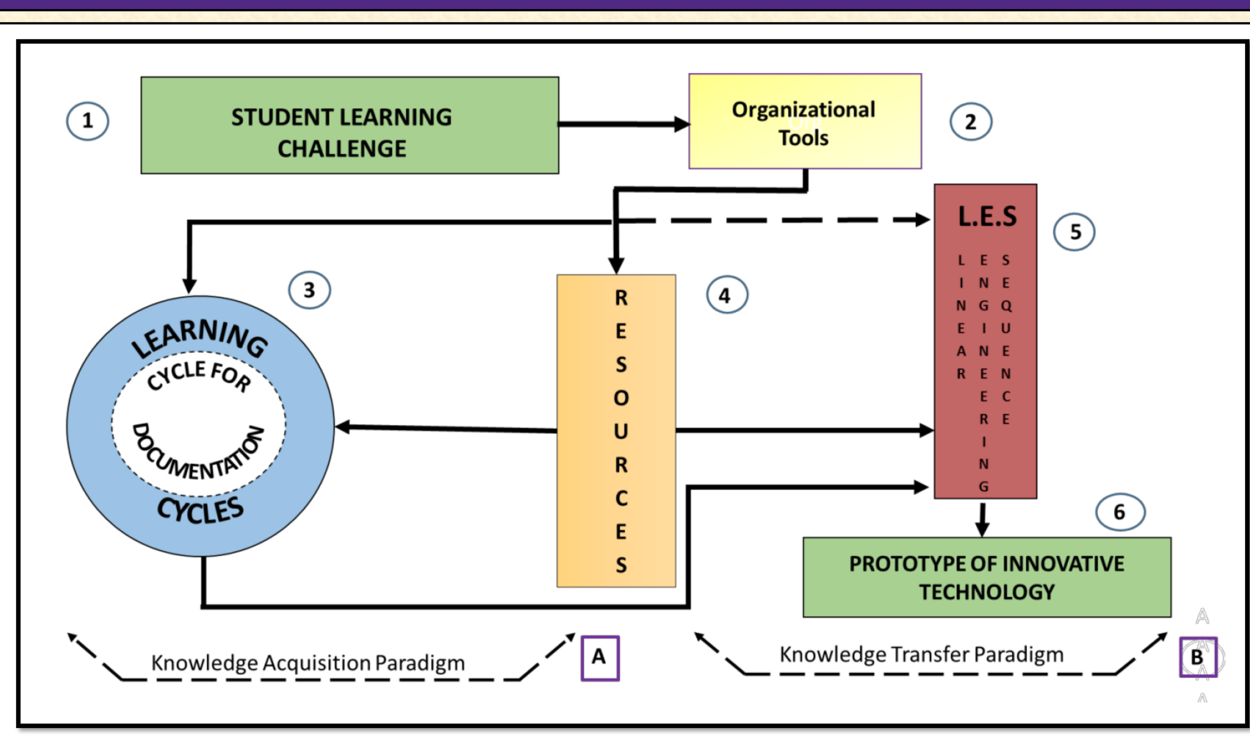
Department of Chemical Engineering; Tennessee Technological University, Cookeville, TN

Motivation and Relevance of Research

Understanding the formulation and the modeling of distinct approaches used in the bio-mathematical foundation to homeostatic wound healing modeling is a critical task to advance the field. In recent contributions (Jorgensen, 2017), researchers have made progress *experimentally* in understanding transport of biomedicines in hydrogels of potential use as an effective scaffolding material to facilitate wound healing. This effort has been complemented by *modelling approaches* (Dawson et. al., 2021)[1] to increase the understanding of the electro-convective diffuse transport of biomolecules in wound healing in electrotherapeutic assisted wound healing applications. In the past, the guiding method of study has been focused on capillaries of cylindrical geometry. This contribution is focused on not only on the area-averaging methodology (Whitaker, 1999) for modeling of the electrostatic potential effects in the wound microenvironment of the scaffolding material, but also on the role that the chosen geometry plays on the electrostatic potential behavior. Therefore, in this study, we are making a comparison of the effects of the electrostatic potential on the microenvironment in two distinct geometries, i.e., cylindrical geometry and the rectangular geometry. Specifically, the impact of the diffusion and the migration of thrombin to induce the conversion of fibrinogen to fibrin will be discussed. Anchored by the Renaissance Foundry Model to guide the overall research, elements of the Electrokinetic-Hydrodynamics will be used to formulate the microscopic scale models that, then, by following an area-averaging algorithm approach will be upscaled to the entire capillary domain. The solutions will be compared analytically and graphically through a set of parametric values corresponding to the voltages applied to the system. Future and ongoing efforts towards this project will be highlighted.

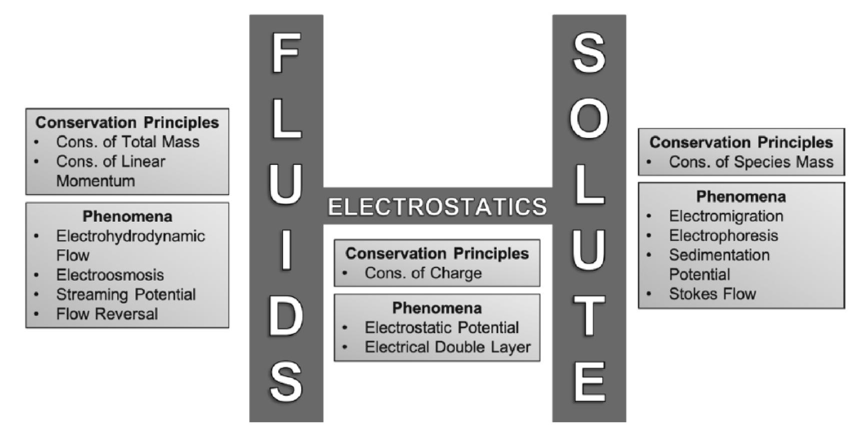
[1]*Electrotherapeutic Assisted Wound Healing: Modeling of the Electrostatic Field in a Porous Gel or Healing Media.* Phoebe Dawson, Steffano Oyanader, Stephanie Jorgensen, Robby Sanders, and Pedro E. Arce

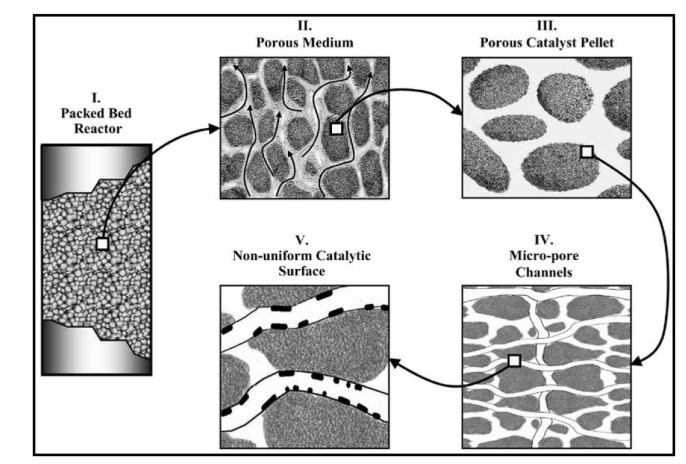
Methodology

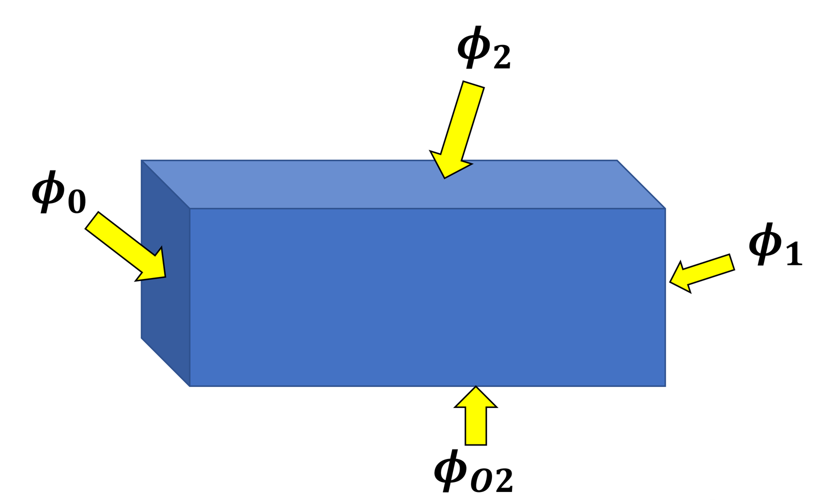


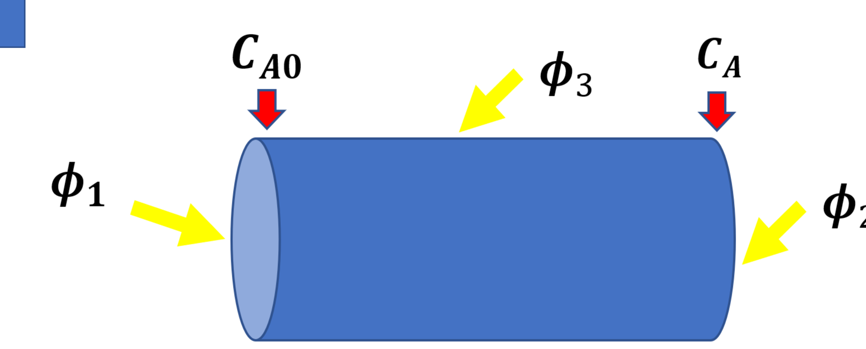
The methodology for modeling the wound microenvironment shown is based on the Renaissance Foundry (see figure above) [3] and the area averaging approach [5]. A thorough review of each of these works help to describe the methodology to solve the transport governing equations used to model the wound microenvironment. In addition, the Electrokinetic-Hydrodynamics H-model [1] will be used to direct the dynamic model.

Development of Study & Model Strategy









Conservation of Electrostatic Charge Cartesian Coordinates

Assumptions	Electrostatic Conservation Equation-RC
- Laplacian Formation - No Effects of the Electrical Field in the y-direction	$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0$$
Boundary Conditions	$\phi_O(z)=K_{01} @ z=0$; $\phi_{02}(y)=K_{O2} @ z=0$; $\phi_1(z)=K_1 @ z=L$; $\phi_2(y)=K_2 @ y=B$

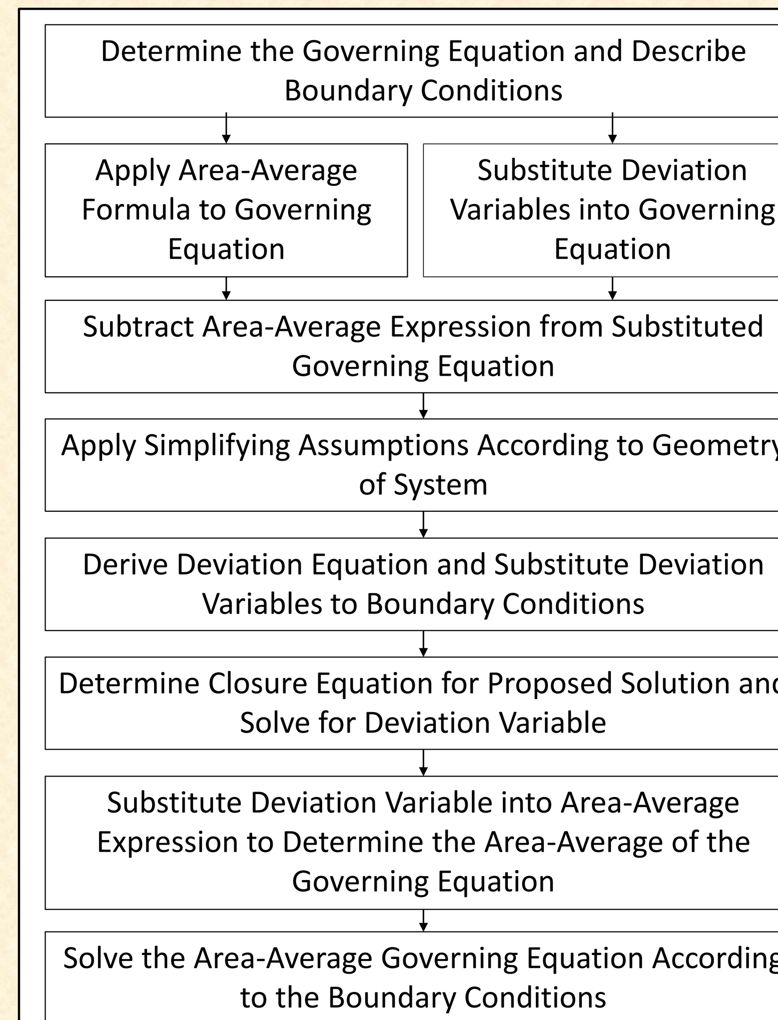
Conservation of Electrostatic Charge Cylindrical Coordinates

Assumptions	Electrostatic Conservation Equation-CC
- Laplacian Formation - No Radial Effects of the Electrical Field	$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^2\phi}{\partial z^2}=0$$
Boundary Conditions	$\frac{\partial\phi}{\partial r}=0 @ r=0$; $\phi(r)=K_1 @ z=0$; $\phi(z)=K_3 @ r=R_0$; $\phi(r)=K_2 @ z=L$

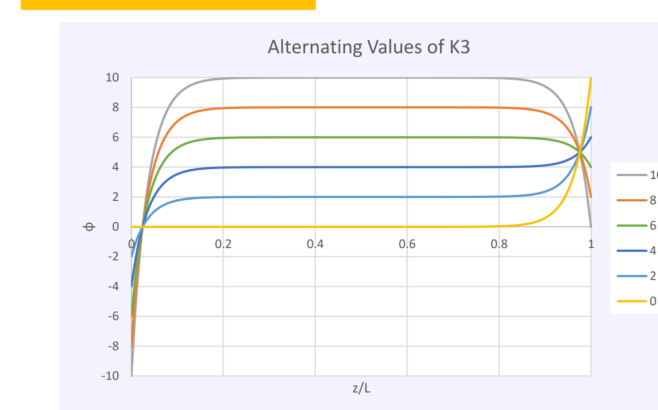
Acknowledgements

We would like to strongly acknowledge Dr. Mario A. Oyanader at California Baptist as a Collaborator for this on-going research project. Additionally, we would also like to acknowledge Kurt Dunham, Sabrina Buer, and Dr. Stephanie Jorgensen for their previous efforts in this work.

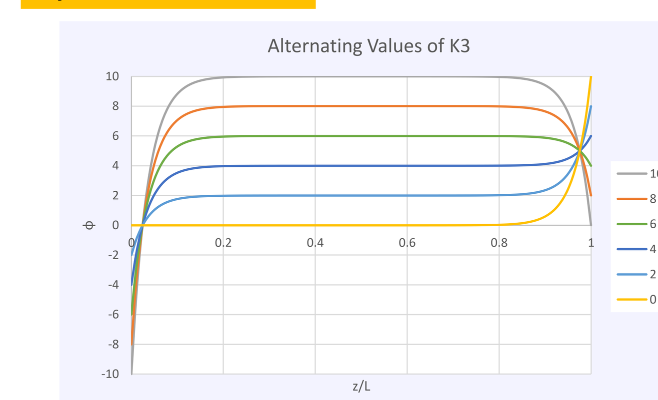
Discussion



Cartesian Coordinates



Cylindrical Coordinates



***Formal Cylindrical Analytical Solution*:**

$$\phi=\langle\phi\rangle+\tilde{\phi}$$

***Formal Average Cylindrical Analytical Solution*:**

$$\tilde{\phi}=K_3+\frac{2*K_3}{R_0^2}*\left(r^2-R_0^2\right)-\left[\frac{4}{R_0^2}*\frac{\left(r^2-R_0^2\right)}{2}+1\right]*\langle\phi\rangle$$

***Formal Cylindrical Analytical Solution*:**

$$\langle\phi\rangle=c_1\cosh\left(\lambda_{cyl}*z\right)+c_2\sinh\left(\lambda_{cyl}*z\right)+K_3$$

Conclusions and Future Work

In conclusion, one can visually compare the solutions provided from the algorithmic approach to solving the Laplace Electrostatic Equation in two separate coordinate systems. A clear and desirable outcome is that not only is there no numerical or visual difference when plotting each of the analytical solutions, but on each of the graphs one can observe a region of the non-dimensional length which behaves as a pseudo-steady state. This is an important scaling effect that will allow for a simplified solution of the Concentration Profile. The Concentration (C_A) will be studied in the future with the effects of the applied electrical field in cartesian and cylindrical coordinates.

Species Continuity Equation in Cylindrical Coordinates:

$$\frac{\partial C_a}{\partial t}=D\frac{\partial^2 C_A}{\partial z^2}+D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right)\right]+z\mu\frac{\partial}{\partial z}\left[C\frac{\partial^2\phi}{\partial z^2}+\frac{\partial C_A}{\partial z}\frac{\partial\phi}{\partial z}\right]$$

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